

Non-linear Theory of Elasticity and Optimal Design

This Page Intentionally Left Blank

Non-linear Theory of Elasticity and Optimal Design

How to build safe economical machines and structures How to build proven reliable physical theory

Leah W. Ratner

2003



ELSEVIER

Amsterdam – Boston – Heidelberg – London – New – York – Oxford Paris – San Diego – San Francisco – Singapore – Sydney – Tokyo ELSEVIER B.V.
Sara Burgerhartstraat 25
P.O. Box 211, 1000 AE Amsterdam, The Netherlands
© 2003 Elsevier B.V. All rights reserved.

This work is protected under copyright by Elsevier, and the following terms and conditions apply to its use:

Photocopying

Single photocopies of single chapters may be made for personal use as allowed by national copyright laws. Permission of the Publisher and payment of a fee is required for all other photocopying, including multiple or systematic copying, copying for advertising or promotional purposes, resale, and all forms of document delivery. Special rates are available for educational institutions that wish to make photocopies for non-profit educational classroom use.

Permissions may be sought directly from Elsevier's Rights Department in Oxford, UK: phone: (+44) 1865 843830, fax (+44) 1865 853333, e-mail: permissions@elsevier.com. Requests may also be completed on-line via the Elsevier home page (http://www.elsevier.com/locate/permissions).

In the USA, users may clear permissions and make payments through the Copyright Clearance Center, Inc., 222 Rosewood Drive, Danvers, MA 01923, USA; phone: (+1) (978) 7508400, fax: (+1) (978) 7504744, and in the UK through the Copyright Licensing Agency Rapid Clearance Service (CLARCS), 90 Tottenham Court Road, London W1P 0LP, UK; phone: (+44) 20 7631 5555; fax: (+44) 20 7631 5500. Other countries may have a local reprographic rights agency for payments.

Derivative Works

Tables of contents may be reproduced for internal circulation, but permission of Elsevier is required for external resale or distribution of such material. Permission of the Publisher is required for all other derivative works, including compilations and translations.

Electronic Storage or Usage

Permission of the Publisher is required to store or use electronically any material contained in this work, including any chapter or part of a chapter.

Except as outlined above, no part of this work may be reproduced, stored in a retrieval system or transmitted in any form or by any means, electronic, mechanical, photocopying, recording or otherwise, without prior written permission of the Publisher.

Address permissions requests to: Elsevier's Rights Department, at the phone, fax and e-mail addresses noted above.

Notice

No responsibility is assumed by the Publisher for any injury and/or damage to persons or property as a matter of products liability, negligence or otherwise, or from any use or operation of any methods, products, instructions or ideas contained in the material herein. Because of rapid advances in the medical sciences, in particular, independent verification of diagnoses and drug dosages should be made.

First edition 2003 ISBN: 0-444-51427-9

Printed in Hungary.

Preface

Contemporary physics from its beginning in the 17th century has been progressing in two parallel directions, i.e. empirical and mathematical. One unresolved problem of epistemology is that these two branches are not really well combined in the scientific theories. Each of them has its role. The empirical methods are used for governing the facts concerning a phenomenon and testing the inferences of a hypothetical theory. Mathematical methods are used for description of the hypothetical physical ideas and making mathematical inferences from these hypotheses. The assumption is made that the mathematical inferences can be tested empirically and that such tests may perhaps not prove but at least validate the theory. Nevertheless empirical validation is insufficient for combining the methods. There is no proof for theories that remain in essence hypothetical. The successful alignment of essentially different methods can be achieved by employing logical structure as a mediating method.

In this work the author proposes logical structure as the frame of a physical theory that allows building a consistent provable theory. The theory presented in this work is a Non-Linear Theory of Elasticity. This theory has a logical frame that makes it a reliable foundation for structural analysis and design.

Part I of the book describes the general principles on which the non-Linear Theory of Elasticity was built. The theory has a new conception of strength and elastic stability of a structure. This part also reproduces the specification of the author's US patent on the method of optimization of structures.

Parts II and III are devoted to the analysis of the current Linear Theory of Elasticity and the new Non-Linear Theory of Elasticity. Part III also analyzes typical structures such as bars, beams, shafts, columns, plates, and shells. The reader will also find there the important discussion on the distribution of elastic forces in a structure and a new hypothesis on the torsion of non-round bars.

vi Preface

Part IV considers some important methodological questions relating to the construction of a theory, such as graph analysis and the geometrical models of physical functions.

Part V discusses philosophical implications of the new methodology in science and discusses in length the definitive logic in the theory of elasticity. The important physical implication of this methodology is the need for a mathematical description of the domain of stable physical relations for a physical phenomenon.

Contents

Preface Introduction Prologue					
			Par	t I. Principles and Methods of NLTE	11
			1.	Practical problems	11
2.	Foundations of the non-linear theory of elasticity	12			
	2.1. Summary	20			
	2.2. Recapture	21			
3.	Devising the non-linear theory of elasticity	22			
	3.1. Summary	28			
4.	Principles of logic in NLTE	29			
5.	Method of optimal structural design	44			
	5.1. Summary	48			
	5.2. Example of beam design	49			
6.	Optimal structural design (examples)	50			
	6.1. Tension/compression and bending	50			
	6.2. Beams with multiple supports	50			
	6.3. Deformation of plates	50			
	Optimal simple beam	51			
8.	On mathematics in physics	52			
	8.1. Summary	58			
9.	On the nature of the limit of elasticity	59			
	9.1. Summary	61			
	The stress–strain diagram	61			
11.	On the nature of proof in physical theory	62			
	11.1. Summary	64			
12.	History of the theory of elasticity	64			

viii Contents

13.	On the principles of the theory of elasticity
Uni	ited States Patent 5,654,900 (August 5, 1997)
	ethod of and Apparatus for Optimization of Structures.
	Background of the invention
1.	1.1. Field of the Invention
	1.2. Description of the Prior Art
2.	Summary of the invention
	Description of illustrated exemplary teaching
	- see-passe of translation of the passes of
Par	t II. Linear Theory of Infinitesimal Deformations
	Principles of LTE
	Stress
3.	Deformation
4.	Hooke's Law
5.	Geometric characteristics of plane areas
6.	Combination of stresses
	6.1. Load and Resistance Factor Design (LRFD)
_	
	t III. Optimization of typical structures
	Introduction
	Tension/compression
3.	Torsion
4	3.1. Recapture
4:	Bending
_	4.1. Calculation of deflections using the unit load method.
	Combined stresses
6.	
/.	Stability of thin shells
Ω	7.1. Calculation for symmetrical thin shells
	Elastic stability of plates
	Dynamic stresses and the non-linear theory of elasticity
10.	1
	10.1. Tension impact on a bar10.2. Bending impact
	10.2. Dending impact

Contents ix

Appendix I. Optimal design of typical beams Appendix II Tension-compression Bending Circular cylindrical shells (membrane theory) Appendix III. Table for shaft calculation Part IV. Further Discussions in the Theory of Elasticity 1. Graph analysis 1.1. Commentary to Illustration 1 of Part I 2. Geometrical models of physical functions 3. The equation for the elastic line and the non-linear theory of elasticity Part V. Philosophy and Logic of Physical Theory 1. Philosophical background of the non-linear theory of elasticity 2. Logic and physical theory 2.1. Role of logic in science 2.2. General argument 3. The rules of logic 4. Logic of construction in NLTE 5. The definitive logic 5.1. Recapture 6. It is possible to prove physical theory 7. Notes on logic 7.1. Commentaries to "Preface to Logic" by Morris R. Cohen 7.2. Commentaries to "An Introduction to the Philosophy of Science" by Rudolf Carnap 7.2.1. Definition of scientific law 7.2.2. Induction	11. Testing of materials				
Appendix II Tension—compression Bending Circular cylindrical shells (membrane theory) Appendix III. Table for shaft calculation Part IV. Further Discussions in the Theory of Elasticity 1. Graph analysis 1.1. Commentary to Illustration 1 of Part I 2. Geometrical models of physical functions 3. The equation for the elastic line and the non-linear theory of elasticity Part V. Philosophy and Logic of Physical Theory 1. Philosophical background of the non-linear theory of elasticity 2. Logic and physical theory 2.1. Role of logic in science 2.2. General argument 3. The rules of logic 4. Logic of construction in NLTE 5. The definitive logic 5.1. Recapture 6. It is possible to prove physical theory 7. Notes on logic 7.1. Commentaries to "Preface to Logic" by Morris R. Cohen 7.2. Commentaries to "An Introduction to the Philosophy of Science" by Rudolf Carnap 7.2.1. Definition of scientific law 7.2.2. Induction					
Tension—compression Bending Circular cylindrical shells (membrane theory) Appendix III. Table for shaft calculation Part IV. Further Discussions in the Theory of Elasticity 1. Graph analysis 1.1. Commentary to Illustration 1 of Part I 2. Geometrical models of physical functions 3. The equation for the elastic line and the non-linear theory of elasticity Part V. Philosophy and Logic of Physical Theory 1. Philosophical background of the non-linear theory of elasticity 2. Logic and physical theory 2.1. Role of logic in science 2.2. General argument 3. The rules of logic 4. Logic of construction in NLTE 5. The definitive logic 5.1. Recapture 6. It is possible to prove physical theory 7. Notes on logic 7.1. Commentaries to "Preface to Logic" by Morris R. Cohen 7.2. Commentaries to "An Introduction to the Philosophy of Science" by Rudolf Carnap 7.2.1. Definition of scientific law 7.2.2. Induction					
Bending Circular cylindrical shells (membrane theory) Appendix III. Table for shaft calculation Part IV. Further Discussions in the Theory of Elasticity 1. Graph analysis 1.1. Commentary to Illustration 1 of Part I 2. Geometrical models of physical functions 3. The equation for the elastic line and the non-linear theory of elasticity Part V. Philosophy and Logic of Physical Theory 1. Philosophical background of the non-linear theory of elasticity 2. Logic and physical theory 2.1. Role of logic in science 2.2. General argument 3. The rules of logic 4. Logic of construction in NLTE 5. The definitive logic 5.1. Recapture 6. It is possible to prove physical theory 7. Notes on logic 7.1. Commentaries to "Preface to Logic" by Morris R. Cohen 7.2. Commentaries to "An Introduction to the Philosophy of Science" by Rudolf Carnap 7.2.1. Definition of scientific law 7.2.2. Induction					
Circular cylindrical shells (membrane theory) Appendix III. Table for shaft calculation Part IV. Further Discussions in the Theory of Elasticity 1. Graph analysis 1.1. Commentary to Illustration 1 of Part I 2. Geometrical models of physical functions 3. The equation for the elastic line and the non-linear theory of elasticity Part V. Philosophy and Logic of Physical Theory 1. Philosophical background of the non-linear theory of elasticity 2. Logic and physical theory 2.1. Role of logic in science 2.2. General argument 3. The rules of logic 4. Logic of construction in NLTE 5. The definitive logic 5.1. Recapture 6. It is possible to prove physical theory 7. Notes on logic 7.1. Commentaries to "Preface to Logic" by Morris R. Cohen 7.2. Commentaries to "An Introduction to the Philosophy of Science" by Rudolf Carnap 7.2.1. Definition of scientific law 7.2.2. Induction					
Appendix III. Table for shaft calculation Part IV. Further Discussions in the Theory of Elasticity 1. Graph analysis 1.1. Commentary to Illustration 1 of Part I 2. Geometrical models of physical functions 3. The equation for the elastic line and the non-linear theory of elasticity Part V. Philosophy and Logic of Physical Theory 1. Philosophical background of the non-linear theory of elasticity 2. Logic and physical theory 2.1. Role of logic in science 2.2. General argument 3. The rules of logic 4. Logic of construction in NLTE 5. The definitive logic 5.1. Recapture 6. It is possible to prove physical theory 7. Notes on logic 7.1. Commentaries to "Preface to Logic" by Morris R. Cohen 7.2. Commentaries to "An Introduction to the Philosophy of Science" by Rudolf Carnap 7.2.1. Definition of scientific law 7.2.2. Induction					
Part IV. Further Discussions in the Theory of Elasticity 1. Graph analysis 1.1. Commentary to Illustration 1 of Part I 2. Geometrical models of physical functions 3. The equation for the elastic line and the non-linear theory of elasticity Part V. Philosophy and Logic of Physical Theory 1. Philosophical background of the non-linear theory of elasticity. 2. Logic and physical theory 2.1. Role of logic in science 2.2. General argument 3. The rules of logic 4. Logic of construction in NLTE 5. The definitive logic 5.1. Recapture 6. It is possible to prove physical theory 7. Notes on logic 7.1. Commentaries to "Preface to Logic" by Morris R. Cohen 7.2. Commentaries to "An Introduction to the Philosophy of Science" by Rudolf Carnap 7.2.1. Definition of scientific law 7.2.2. Induction					
1.1. Commentary to Illustration 1 of Part I 2. Geometrical models of physical functions 3. The equation for the elastic line and the non-linear theory of elasticity Part V. Philosophy and Logic of Physical Theory 1. Philosophical background of the non-linear theory of elasticity 2. Logic and physical theory 2.1. Role of logic in science 2.2. General argument 3. The rules of logic 4. Logic of construction in NLTE 5. The definitive logic 5.1. Recapture 6. It is possible to prove physical theory 7. Notes on logic 7.1. Commentaries to "Preface to Logic" by Morris R. Cohen 7.2. Commentaries to "An Introduction to the Philosophy of Science" by Rudolf Carnap 7.2.1. Definition of scientific law 7.2.2. Induction	Appendix III. Tuble for shall calculation				
1.1. Commentary to Illustration 1 of Part I 2. Geometrical models of physical functions 3. The equation for the elastic line and the non-linear theory of elasticity Part V. Philosophy and Logic of Physical Theory 1. Philosophical background of the non-linear theory of elasticity 2. Logic and physical theory 2.1. Role of logic in science 2.2. General argument 3. The rules of logic 4. Logic of construction in NLTE 5. The definitive logic 5.1. Recapture 6. It is possible to prove physical theory 7. Notes on logic 7.1. Commentaries to "Preface to Logic" by Morris R. Cohen 7.2. Commentaries to "An Introduction to the Philosophy of Science" by Rudolf Carnap 7.2.1. Definition of scientific law 7.2.2. Induction	Part IV. Further Discussions in the Theory of Elasticity				
1.1. Commentary to Illustration 1 of Part I 2. Geometrical models of physical functions 3. The equation for the elastic line and the non-linear theory of elasticity Part V. Philosophy and Logic of Physical Theory 1. Philosophical background of the non-linear theory of elasticity 2. Logic and physical theory 2.1. Role of logic in science 2.2. General argument 3. The rules of logic 4. Logic of construction in NLTE 5. The definitive logic 5.1. Recapture 6. It is possible to prove physical theory 7. Notes on logic 7.1. Commentaries to "Preface to Logic" by Morris R. Cohen 7.2. Commentaries to "An Introduction to the Philosophy of Science" by Rudolf Carnap 7.2.1. Definition of scientific law 7.2.2. Induction	· ·				
2. Geometrical models of physical functions 3. The equation for the elastic line and the non-linear theory of elasticity Part V. Philosophy and Logic of Physical Theory 1. Philosophical background of the non-linear theory of elasticity. 2. Logic and physical theory 2.1. Role of logic in science 2.2. General argument 3. The rules of logic. 4. Logic of construction in NLTE. 5. The definitive logic 5.1. Recapture 6. It is possible to prove physical theory 7. Notes on logic 7.1. Commentaries to "Preface to Logic" by Morris R. Cohen 7.2. Commentaries to "An Introduction to the Philosophy of Science" by Rudolf Carnap 7.2.1. Definition of scientific law 7.2.2. Induction					
3. The equation for the elastic line and the non-linear theory of elasticity Part V. Philosophy and Logic of Physical Theory 1. Philosophical background of the non-linear theory of elasticity. 2. Logic and physical theory 2.1. Role of logic in science 2.2. General argument 3. The rules of logic 4. Logic of construction in NLTE 5. The definitive logic 5.1. Recapture 6. It is possible to prove physical theory 7. Notes on logic 7.1. Commentaries to "Preface to Logic" by Morris R. Cohen 7.2. Commentaries to "An Introduction to the Philosophy of Science" by Rudolf Carnap 7.2.1. Definition of scientific law 7.2.2. Induction	·				
Part V. Philosophy and Logic of Physical Theory 1. Philosophical background of the non-linear theory of elasticity. 2. Logic and physical theory 2.1. Role of logic in science 2.2. General argument 3. The rules of logic. 4. Logic of construction in NLTE. 5. The definitive logic. 5.1. Recapture. 6. It is possible to prove physical theory. 7. Notes on logic. 7.1. Commentaries to "Preface to Logic" by Morris R. Cohen. 7.2. Commentaries to "An Introduction to the Philosophy of Science" by Rudolf Carnap. 7.2.1. Definition of scientific law. 7.2.2. Induction.	<u>*</u> *				
Part V. Philosophy and Logic of Physical Theory 1. Philosophical background of the non-linear theory of elasticity. 2. Logic and physical theory 2.1. Role of logic in science 2.2. General argument 3. The rules of logic 4. Logic of construction in NLTE 5. The definitive logic 5.1. Recapture 6. It is possible to prove physical theory 7. Notes on logic 7.1. Commentaries to "Preface to Logic" by Morris R. Cohen 7.2. Commentaries to "An Introduction to the Philosophy of Science" by Rudolf Carnap 7.2.1. Definition of scientific law 7.2.2. Induction					
 Philosophical background of the non-linear theory of elasticity. Logic and physical theory Role of logic in science General argument The rules of logic. Logic of construction in NLTE. The definitive logic. Recapture. It is possible to prove physical theory. Notes on logic. Commentaries to "Preface to Logic" by Morris R. Cohen. Commentaries to "An Introduction to the Philosophy of Science" by Rudolf Carnap. Definition of scientific law. Induction. 					
 Philosophical background of the non-linear theory of elasticity. Logic and physical theory Role of logic in science General argument The rules of logic. Logic of construction in NLTE. The definitive logic. Recapture. It is possible to prove physical theory. Notes on logic. Commentaries to "Preface to Logic" by Morris R. Cohen. Commentaries to "An Introduction to the Philosophy of Science" by Rudolf Carnap. Definition of scientific law. Induction. 	Part V. Philosophy and Logic of Physical Theory				
elasticity 2. Logic and physical theory 2.1. Role of logic in science 2.2. General argument 3. The rules of logic 4. Logic of construction in NLTE 5. The definitive logic 5.1. Recapture 6. It is possible to prove physical theory 7. Notes on logic 7.1. Commentaries to "Preface to Logic" by Morris R. Cohen 7.2. Commentaries to "An Introduction to the Philosophy of Science" by Rudolf Carnap 7.2.1. Definition of scientific law 7.2.2. Induction					
 Logic and physical theory Role of logic in science General argument The rules of logic. Logic of construction in NLTE. The definitive logic. Recapture. It is possible to prove physical theory. Notes on logic. Commentaries to "Preface to Logic" by Morris R. Cohen. Commentaries to "An Introduction to the Philosophy of Science" by Rudolf Carnap. Definition of scientific law. Induction. 	•				
 2.1. Role of logic in science 2.2. General argument 3. The rules of logic 4. Logic of construction in NLTE 5. The definitive logic 5.1. Recapture 6. It is possible to prove physical theory 7. Notes on logic 7.1. Commentaries to "Preface to Logic" by Morris R. Cohen 7.2. Commentaries to "An Introduction to the Philosophy of Science" by Rudolf Carnap 7.2.1. Definition of scientific law 7.2.2. Induction 					
 2.2. General argument 3. The rules of logic 4. Logic of construction in NLTE 5. The definitive logic 5.1. Recapture 6. It is possible to prove physical theory 7. Notes on logic 7.1. Commentaries to "Preface to Logic" by Morris R. Cohen 7.2. Commentaries to "An Introduction to the Philosophy of Science" by Rudolf Carnap 7.2.1. Definition of scientific law 7.2.2. Induction 					
 The rules of logic. Logic of construction in NLTE. The definitive logic. Recapture. It is possible to prove physical theory. Notes on logic. Commentaries to "Preface to Logic" by Morris R. Cohen. Commentaries to "An Introduction to the Philosophy of Science" by Rudolf Carnap. Definition of scientific law. Induction. 					
 Logic of construction in NLTE The definitive logic Recapture It is possible to prove physical theory Notes on logic Commentaries to "Preface to Logic" by Morris R. Cohen Commentaries to "An Introduction to the Philosophy of Science" by Rudolf Carnap Definition of scientific law Induction 					
 5. The definitive logic 5.1. Recapture 6. It is possible to prove physical theory 7. Notes on logic 7.1. Commentaries to "Preface to Logic" by Morris R. Cohen 7.2. Commentaries to "An Introduction to the Philosophy of Science" by Rudolf Carnap 7.2.1. Definition of scientific law 7.2.2. Induction 					
 5.1. Recapture 6. It is possible to prove physical theory 7. Notes on logic 7.1. Commentaries to "Preface to Logic" by Morris R. Cohen 7.2. Commentaries to "An Introduction to the Philosophy of Science" by Rudolf Carnap 7.2.1. Definition of scientific law 7.2.2. Induction 					
 6. It is possible to prove physical theory 7. Notes on logic 7.1. Commentaries to "Preface to Logic" by Morris R. Cohen 7.2. Commentaries to "An Introduction to the Philosophy of Science" by Rudolf Carnap 7.2.1. Definition of scientific law 7.2.2. Induction 					
 7. Notes on logic 7.1. Commentaries to "Preface to Logic" by Morris R. Cohen 7.2. Commentaries to "An Introduction to the Philosophy of Science" by Rudolf Carnap 7.2.1. Definition of scientific law 7.2.2. Induction 	-				
 7.1. Commentaries to "Preface to Logic" by Morris R. Cohen 7.2. Commentaries to "An Introduction to the Philosophy of Science" by Rudolf Carnap 7.2.1. Definition of scientific law 7.2.2. Induction 					
R. Cohen 7.2. Commentaries to "An Introduction to the Philosophy of Science" by Rudolf Carnap 7.2.1. Definition of scientific law 7.2.2. Induction					
7.2. Commentaries to "An Introduction to the Philosophy of Science" by Rudolf Carnap	7.1. Commentaries to "Preface to Logic" by Morris				
of Science" by Rudolf Carnap	R. Cohen				
7.2.1. Definition of scientific law	7.2. Commentaries to "An Introduction to the Philosophy				
7.2.1. Definition of scientific law	of Science" by Rudolf Carnap				
7.2.2. Induction					
/.2.3. Concepts in science	7.2.3. Concepts in science				
7.2.4. Measurement					
7.2.5. Geometry and a theory					

x Contents

	7.2.6. Kant's Synthetic a Priori
	7.3. Notes on methodology of science
	7.4. On the nature of a scientific theory
	7.5. The theory of elasticity as an organized knowledge
	7.6. Logic in mathematics. Commentaries to Bertrand
	Russell and Kurt Gödel
	7.7. On explanation of a physical theory
	7.7.1. Inferential conception of explanation
	7.7.2. The causal conception of scientific explanation.
	7.7.3. The erotetic conception of scientific explanation
	7.8. Theory and observation
	7.9. Validation of scientific theory
	7.9.1. Justificationism
	7.9.2. Falsificationism
	7.9.3. Conventionalism
	7.9.4. The methodology of scientific research
	programmes
	7.9.5. The testing paradigm of scientific inference
	7.9.6. Summary
	7.10. On the logic of truth-function
	7.11. On the logic of classes
	7.11.1. On the logicist systems
8.	Conclusion
9.	Recapture of the central ideas
Bibl	liography
Sub	ject Index