# NON-LINEAR THEORY OF <br> ELASTICITY 

AND OPTIMAL DESIGN
L.W. RATNER

Non-linear Theory of Elasticity and Optimal Design

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# Non-linear Theory of Elasticity and Optimal Design 

How to build safe economical machines and structures How to build proven reliable physical theory

## Leah W. Ratner

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## Preface

Contemporary physics from its beginning in the $17^{\text {th }}$ century has been progressing in two parallel directions, i.e. empirical and mathematical. One unresolved problem of epistemology is that these two branches are not really well combined in the scientific theories. Each of them has its role. The empirical methods are used for governing the facts concerning a phenomenon and testing the inferences of a hypothetical theory. Mathematical methods are used for description of the hypothetical physical ideas and making mathematical inferences from these hypotheses. The assumption is made that the mathematical inferences can be tested empirically and that such tests may perhaps not prove but at least validate the theory. Nevertheless empirical validation is insufficient for combining the methods. There is no proof for theories that remain in essence hypothetical. The successful alignment of essentially different methods can be achieved by employing logical structure as a mediating method.

In this work the author proposes logical structure as the frame of a physical theory that allows building a consistent provable theory. The theory presented in this work is a Non-Linear Theory of Elasticity. This theory has a logical frame that makes it a reliable foundation for structural analysis and design.

Part I of the book describes the general principles on which the nonLinear Theory of Elasticity was built. The theory has a new conception of strength and elastic stability of a structure. This part also reproduces the specification of the author's US patent on the method of optimization of structures.

Parts II and III are devoted to the analysis of the current Linear Theory of Elasticity and the new Non-Linear Theory of Elasticity. Part III also analyzes typical structures such as bars, beams, shafts, columns, plates, and shells. The reader will also find there the important discussion on the distribution of elastic forces in a structure and a new hypothesis on the torsion of non-round bars.

Part IV considers some important methodological questions relating to the construction of a theory, such as graph analysis and the geometrical models of physical functions.

Part V discusses philosophical implications of the new methodology in science and discusses in length the definitive logic in the theory of elasticity. The important physical implication of this methodology is the need for a mathematical description of the domain of stable physical relations for a physical phenomenon.

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