



SENSITIVITY ANALYSIS IN PRACTICE

A Guide to Assessing Scientific Models

ANDREA SALTELLI
STEFANO TARANTOLA
FRANCESCA CAMPOLONGO
MARCO RATTO

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**Andrea Saltelli, Stefano Tarantola,
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Joint Research Centre of the European Commission, Ispra, Italy



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CONTENTS

PREFACE	ix
1 A WORKED EXAMPLE	1
1.1 A simple model	1
1.2 Modulus version of the simple model	10
1.3 Six-factor version of the simple model	15
1.4 The simple model ‘by groups’	22
1.5 The (less) simple correlated-input model	25
1.6 Conclusions	28
2 GLOBAL SENSITIVITY ANALYSIS FOR IMPORTANCE ASSESSMENT	31
2.1 Examples at a glance	31
2.2 What is sensitivity analysis?	42
2.3 Properties of an ideal sensitivity analysis method	47
2.4 Defensible settings for sensitivity analysis	49
2.5 Caveats	56
3 TEST CASES	63
3.1 The jumping man. Applying variance-based methods	63
3.2 Handling the risk of a financial portfolio: the problem of hedging. Applying Monte Carlo filtering and variance-based methods	66
3.3 A model of fish population dynamics. Applying the method of Morris	71
3.4 The Level E model. Radionuclide migration in the geosphere. Applying variance-based methods and Monte Carlo filtering	77
3.5 Two spheres. Applying variance based methods in estimation/calibration problems	83
3.6 A chemical experiment. Applying variance based methods in estimation/calibration problems	85
3.7 An analytical example. Applying the method of Morris	88

4	THE SCREENING EXERCISE	91
4.1	Introduction	91
4.2	The method of Morris	94
4.3	Implementing the method	100
4.4	Putting the method to work: an analytical example	103
4.5	Putting the method to work: sensitivity analysis of a fish population model	104
4.6	Conclusions	107
5	METHODS BASED ON DECOMPOSING THE VARIANCE OF THE OUTPUT	109
5.1	The settings	109
5.2	Factors Prioritisation Setting	110
5.3	First-order effects and interactions	111
5.4	Application of S_i to Setting 'Factors Prioritisation'	112
5.5	More on variance decompositions	118
5.6	Factors Fixing (FF) Setting	120
5.7	Variance Cutting (VC) Setting	121
5.8	Properties of the variance based methods	123
5.9	How to compute the sensitivity indices: the case of orthogonal input	124
5.9.1	A digression on the Fourier Amplitude Sensitivity Test (FAST)	132
5.10	How to compute the sensitivity indices: the case of non-orthogonal input	132
5.11	Putting the method to work: the Level E model	136
5.11.1	Case of orthogonal input factors	137
5.11.2	Case of correlated input factors	144
5.12	Putting the method to work: the bungee jumping model	145
5.13	Caveats	148
6	SENSITIVITY ANALYSIS IN DIAGNOSTIC MODELLING: MONTE CARLO FILTERING AND REGIONALISED SENSITIVITY ANALYSIS, BAYESIAN UNCERTAINTY ESTIMATION AND GLOBAL SENSITIVITY ANALYSIS	151
6.1	Model calibration and Factors Mapping Setting	151
6.2	Monte Carlo filtering and regionalised sensitivity analysis	153
6.2.1	Caveats	155
6.3	Putting MC filtering and RSA to work: the problem of hedging a financial portfolio	161
6.4	Putting MC filtering and RSA to work: the Level E test case	167

6.5	Bayesian uncertainty estimation and global sensitivity analysis	170
6.5.1	Bayesian uncertainty estimation	170
6.5.2	The GLUE case	173
6.5.3	Using global sensitivity analysis in the Bayesian uncertainty estimation	175
6.5.4	Implementation of the method	178
6.6	Putting Bayesian analysis and global SA to work: two spheres	178
6.7	Putting Bayesian analysis and global SA to work: a chemical experiment	184
6.7.1	Bayesian uncertainty analysis (GLUE case)	185
6.7.2	Global sensitivity analysis	185
6.7.3	Correlation analysis	188
6.7.4	Further analysis by varying temperature in the data set: fewer interactions in the model	189
6.8	Caveats	191
7	HOW TO USE SIMLAB	193
7.1	Introduction	193
7.2	How to obtain and install SIMLAB	194
7.3	SIMLAB main panel	194
7.4	Sample generation	197
7.4.1	FAST	198
7.4.2	Fixed sampling	198
7.4.3	Latin hypercube sampling (LHS)	198
7.4.4	The method of Morris	199
7.4.5	Quasi-Random LpTau	199
7.4.6	Random	200
7.4.7	Replicated Latin Hypercube (r-LHS)	200
7.4.8	The method of Sobol'	200
7.4.9	How to induce dependencies in the input factors	200
7.5	How to execute models	201
7.6	Sensitivity analysis	202
8	FAMOUS QUOTES: SENSITIVITY ANALYSIS IN THE SCIENTIFIC DISCOURSE	205
	REFERENCES	211
	INDEX	217

PREFACE

This book is a ‘primer’ in global sensitivity analysis (SA). Its ambition is to enable the reader to apply global SA to a mathematical or computational model. It offers a description of a few selected techniques for sensitivity analysis, used for assessing the relative importance of model input factors. These techniques will answer questions of the type ‘which of the uncertain input factors is more important in determining the uncertainty in the output of interest?’ or ‘if we could eliminate the uncertainty in one of the input factors, which factor should we choose to reduce the most the variance of the output?’ Throughout this primer, the input factors of interest will be those that are uncertain, i.e. whose value lie within a finite interval of non-zero width. As a result, the reader will not find sensitivity analysis methods here that look at the local property of the input–output relationships, such as derivative-based analysis¹. Special attention is paid to the selection of the method, to the framing of the analysis and to the interpretation and presentation of the results. The examples will help the reader to apply the methods in a way that is unambiguous and justifiable, so as to make the sensitivity analysis an added value to model-based studies or assessments. Both diagnostic and prognostic uses of models will be considered (a description of these is in Chapter 2), and Bayesian tools of analysis will be applied in conjunction with sensitivity analysis. When discussing sensitivity with respect to factors, we shall interpret the term ‘factor’ in a very broad sense: a factor is anything that can be changed in a model prior to its execution. This also includes structural or epistemic sources of uncertainty. To make an example, factors will be presented in applications that are in fact ‘triggers’, used to select one model structure versus another, one mesh size versus another, or altogether different conceptualisations of a system.

¹ A cursory exception is in Chapter 1.

Often, models use multi-dimensional uncertain parameters and/or input data to define the geographically distributed properties of a natural system. In such cases, a reduced set of scalar factors has to be identified in order to characterise the multi-dimensional uncertainty in a condensed, but exhaustive fashion. Factors will be sampled either from their prior distribution, or from their posterior distribution, if this is available. The main methods that we present in this primer are all related to one another and are the method of Morris for factors' screening and variance-based measures². Also touched upon are Monte Carlo filtering in conjunction with either a variance based method or a simple two-sample test such as the Smirnov test. All methods used in this book are model-free, in the sense that their application does not rely on special assumptions on the behaviour of the model (such as linearity, monotonicity and additivity of the relationship between input factors and model output).

The reader is encouraged to replicate the test cases offered in this book before trying the methods on the model of interest. To this effect, the SIMLAB software for sensitivity analysis is offered. It is available free on the Web-page of this book <http://www.jrc.cec.eu.int/uasa/primer-SA.asp>. Also available at the same URL are a set of scripts in MATLAB[®] and the GLUEWIN software that implements a combination of global sensitivity analysis, Monte Carlo filtering and Bayesian uncertainty estimation.

This book is organised as follows. The first chapter presents the reader with most of the main concepts of the book, through their application to a simple example, and offers boxes with recipes to replicate the example using SIMLAB. All the concepts will then be revisited in the subsequent chapters. In Chapter 2 we offer another preview of the contents of the book, introducing succinctly the examples and their role in the primer. Chapter 2 also gives some definitions of the subject matter and ideas about the framing of the sensitivity analysis in relation to the defensibility of model-based assessment. Chapter 3 gives a full description of the test cases. Chapter 4 tackles screening methods for

² Variance based measures are generally estimated numerically using either the method of Sobol' or FAST (Fourier Analysis Sensitivity Test), or extensions of these methods available in the SIMLAB software that comes with this primer.

sensitivity analysis, and in particular the method of Morris, with applications. Chapter 5 discusses variance based measures, with applications. More ideas about ‘setting for the analysis’ are presented here. Chapter 6 covers Bayesian uncertainty estimation and Monte Carlo filtering, with emphasis on the links with global sensitivity analysis. Chapter 7 gives some instructions on how to use SIMLAB and, finally, Chapter 8 gives a few concepts and some opinions of various practitioners about SA and its implication for an epistemology of model use in the scientific discourse.