



SENSITIVITY ANALYSIS IN PRACTICE

A Guide to Assessing Scientific Models

ANDREA SALTELLI
STEFANO TARANTOLA
FRANCESCA CAMPOLONGO
MARCO RATTO

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SCIENTIFIC MODELS

**Andrea Saltelli, Stefano Tarantola,
Francesca Campolongo and Marco Ratto**

Joint Research Centre of the European Commission, Ispra, Italy



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PREFACE

This book is a ‘primer’ in global sensitivity analysis (SA). Its ambition is to enable the reader to apply global SA to a mathematical or computational model. It offers a description of a few selected techniques for sensitivity analysis, used for assessing the relative importance of model input factors. These techniques will answer questions of the type ‘which of the uncertain input factors is more important in determining the uncertainty in the output of interest?’ or ‘if we could eliminate the uncertainty in one of the input factors, which factor should we choose to reduce the most the variance of the output?’ Throughout this primer, the input factors of interest will be those that are uncertain, i.e. whose value lie within a finite interval of non-zero width. As a result, the reader will not find sensitivity analysis methods here that look at the local property of the input–output relationships, such as derivative-based analysis¹. Special attention is paid to the selection of the method, to the framing of the analysis and to the interpretation and presentation of the results. The examples will help the reader to apply the methods in a way that is unambiguous and justifiable, so as to make the sensitivity analysis an added value to model-based studies or assessments. Both diagnostic and prognostic uses of models will be considered (a description of these is in Chapter 2), and Bayesian tools of analysis will be applied in conjunction with sensitivity analysis. When discussing sensitivity with respect to factors, we shall interpret the term ‘factor’ in a very broad sense: a factor is anything that can be changed in a model prior to its execution. This also includes structural or epistemic sources of uncertainty. To make an example, factors will be presented in applications that are in fact ‘triggers’, used to select one model structure versus another, one mesh size versus another, or altogether different conceptualisations of a system.

¹ A cursory exception is in Chapter 1.

Often, models use multi-dimensional uncertain parameters and/or input data to define the geographically distributed properties of a natural system. In such cases, a reduced set of scalar factors has to be identified in order to characterise the multi-dimensional uncertainty in a condensed, but exhaustive fashion. Factors will be sampled either from their prior distribution, or from their posterior distribution, if this is available. The main methods that we present in this primer are all related to one another and are the method of Morris for factors' screening and variance-based measures². Also touched upon are Monte Carlo filtering in conjunction with either a variance based method or a simple two-sample test such as the Smirnov test. All methods used in this book are model-free, in the sense that their application does not rely on special assumptions on the behaviour of the model (such as linearity, monotonicity and additivity of the relationship between input factors and model output).

The reader is encouraged to replicate the test cases offered in this book before trying the methods on the model of interest. To this effect, the SIMLAB software for sensitivity analysis is offered. It is available free on the Web-page of this book <http://www.jrc.cec.eu.int/uasa/primer-SA.asp>. Also available at the same URL are a set of scripts in MATLAB[®] and the GLUEWIN software that implements a combination of global sensitivity analysis, Monte Carlo filtering and Bayesian uncertainty estimation.

This book is organised as follows. The first chapter presents the reader with most of the main concepts of the book, through their application to a simple example, and offers boxes with recipes to replicate the example using SIMLAB. All the concepts will then be revisited in the subsequent chapters. In Chapter 2 we offer another preview of the contents of the book, introducing succinctly the examples and their role in the primer. Chapter 2 also gives some definitions of the subject matter and ideas about the framing of the sensitivity analysis in relation to the defensibility of model-based assessment. Chapter 3 gives a full description of the test cases. Chapter 4 tackles screening methods for

² Variance based measures are generally estimated numerically using either the method of Sobol' or FAST (Fourier Analysis Sensitivity Test), or extensions of these methods available in the SIMLAB software that comes with this primer.

sensitivity analysis, and in particular the method of Morris, with applications. Chapter 5 discusses variance based measures, with applications. More ideas about ‘setting for the analysis’ are presented here. Chapter 6 covers Bayesian uncertainty estimation and Monte Carlo filtering, with emphasis on the links with global sensitivity analysis. Chapter 7 gives some instructions on how to use SIMLAB and, finally, Chapter 8 gives a few concepts and some opinions of various practitioners about SA and its implication for an epistemology of model use in the scientific discourse.