

Introduction to
**Quantum
Mechanics**

A. C. Phillips

The cover features a dark blue background with a pattern of lighter blue circles. Two large circles are highlighted with a red glow. One is a solid red circle, and the other is a red circle with a black interior containing several smaller red circles, resembling a quantum dot or a lattice structure.

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Introduction to Quantum Mechanics

The Manchester Physics Series

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INTRODUCTION TO QUANTUM MECHANICS

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To my sons:

Joseph
Michael
Patrick
Peter

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Foreword

Sadly, Tony Phillips, a good friend and colleague for more than thirty years, died on 27th November 2002. Over the years, we discussed most topics under the sun. The originality and clarity of his thoughts and the ethical basis of his judgements always made this a refreshing exercise. When discussing physics, quantum mechanics was a recurring theme which gained prominence after his decision to write this book. He completed the manuscript three months before his death and asked me to take care of the proofreading and the Index. A labour of love. I knew what Tony wanted—and what he did not want. Except for corrections, no changes have been made.

Tony was an outstanding teacher who could talk with students of all abilities. He had a deep knowledge of physics and was able to explain subtle ideas in a simple and delightful style. Who else would refer to the end-point of nuclear fusion in the sun as sunshine? Students appreciated him for these qualities, his straightforwardness and his genuine concern for them. This book is a fitting memorial to him.

Franz Mandl
December 2002

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Editors' preface to the Manchester Physics Series

The Manchester Physics Series is a series of textbooks at first degree level. It grew out of our experience at the Department of Physics and Astronomy at Manchester University, widely shared elsewhere, that many textbooks contain much more material than can be accommodated in a typical undergraduate course; and that this material is only rarely so arranged as to allow the definition of a short self-contained course. In planning these books we have had two objectives. One was to produce short books: so that lecturers should find them attractive for undergraduate courses; so that students should not be frightened off by their encyclopaedic size or price. To achieve this, we have been very selective in the choice of topics, with the emphasis on the basic physics together with some instructive, stimulating and useful applications. Our second objective was to produce books which allow courses of different lengths and difficulty to be selected with emphasis on different applications. To achieve such flexibility we have encouraged authors to use flow diagrams showing the logical connections between different chapters and to put some topics in starred sections. These cover more advanced and alternative material which is not required for the understanding of latter parts of each volume.

Although these books were conceived as a series, each of them is self-contained and can be used independently of the others. Several of them are suitable for wider use in other sciences. Each Author's Preface gives details about the level, prerequisites, etc., of that volume.

The Manchester Physics Series has been very successful with total sales of more than a quarter of a million copies. We are extremely grateful to the many students and colleagues, at Manchester and elsewhere, for helpful criticisms and stimulating comments. Our particular thanks go to the authors for all the work they have done, for the many new ideas they have contributed, and for discussing patiently, and often accepting, the suggestions of the editors.

Finally we would like to thank our publishers, John Wiley & Sons, Ltd, for their enthusiastic and continued commitment to the Manchester Physics Series.

D. J. Sandiford

F. Mandl

A. C. Phillips

February 1997

Author's preface

There are many good advanced books on quantum mechanics but there is a distinct lack of books which attempt to give a serious introduction at a level suitable for undergraduates who have a tentative understanding of mathematics, probability and classical physics.

This book introduces the most important aspects of quantum mechanics in the simplest way possible, but challenging aspects which are essential for a meaningful understanding have not been evaded. It is an introduction to quantum mechanics which

- motivates the fundamental postulates of quantum mechanics by considering the weird behaviour of quantum particles
- reviews relevant concepts in classical physics before corresponding concepts are developed in quantum mechanics
- presents mathematical arguments in their simplest form
- provides an understanding of the power and elegance of quantum mechanics that will make more advanced texts accessible.

Chapter 1 provides a qualitative description of the remarkable properties of quantum particles, and these properties are used as the guidelines for a theory of quantum mechanics which is developed in Chapters 2, 3 and 4. Insight into this theory is gained by considering square wells and barriers in Chapter 5 and the harmonic oscillator in Chapter 6. Many of the concepts used in the first six chapters are clarified and developed in Chapter 7. Angular momentum in quantum mechanics is introduced in Chapter 8, but because angular momentum is a demanding topic, this chapter focusses on the ideas that are needed for an understanding of the hydrogen atom in Chapter 9, identical particles in Chapter 10 and many-electron atoms in Chapter 11. Chapter 10 explains why identical particles are described by entangled quantum states and how this entanglement for electrons leads to the Pauli exclusion principle.

Chapters 7 and 10 may be omitted without significant loss of continuity. They deal with concepts which are not needed elsewhere in the book.

I would like to express my thanks to students and colleagues at the University of Manchester. Daniel Guise helpfully calculated the energy levels in a screened Coulomb potential. Thomas York used his impressive computing skills to provide representations of the position probabilities for particles with different orbital angular momentum. Sean Freeman read an early version of the first six chapters and provided suggestions and encouragement. Finally, I would like to thank Franz Mandl for reading an early version of the book and for making forcefully intelligent suggestions for improvement.

A. C. Phillips

August 2002