MINISTRY OF EDUCATION AND TRAINING

DANANG UNIVERSITY

LE THI THU NGA

RESEARCH ON PARAMETRIC SURFACE MODELING FROM A 3D MESH

Specialty: Computer science
Code: 62.48.01.01

SUMMARY OF DOCTORAL THESIS

Danang - 2018

The doctoral thesis has been finished at

Danang University

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Time: 08h30.

Date: 19/05/2018.

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INTRODUCTION

The 3D geometric models play an important role for modeling, designing and reconstructing the surface of real objects on a computer. Nowadays, the models have been widely used in many fields of computer graphic, animation, game 3D, computer aided geometric design, reverse engineering, virtual reality; as well as physics, geology, medicine, chemistry, etc. It is necessary to display and process these models correctly and quickly in the practical applications.

Almost objects are shown on a computer using polygon mesh model. This model is able to display and shade the object surfaces faster and more effectively. However, the polygon mesh also has limitations such as: cannot possibly distinguish the visible or invisible object areas, cannot realize the type of curves, difficult to evaluate exactly a point on the object surface, cannot possibly find out collisions between objects, and difficult to calculate physical properties, etc.

Whereas parametric surface model not only displays surfaces with smoothness, high continuity, steadiness, flexibility and local control capacity by control points of the surface; but also has operations, algorithms to compute any point position on the surface correctly and efficiently. Therefore, the parametric surface is suitable to construct 3D models, shade, display or represent the object surface on a computer. They efficiently support applications which are capable of interacting the object surface. For example, modeling virtual object, detecting collision and deformation of surface, calculating normal-force in virtual reality (VR); reconstructing surface in reverse engineering (RE); mapping texture, morphing in computer graphic (CG); modeling surface of topography, determining height and fault in geographic information system (GIS); computing physical characteristics of object such as weight, surface area, volume, centroid; calculating tension, heat transfer and fluid dynamics in the finite element method (FEM), etc.

It is really necessary to exploit the advantages of both models in the modeling applications. Therefore, converting between polygon mesh and parametric surface for performative and interaction is a sciential concern and has many applications in potential industries. The aim of the thesis is to create a model for reconstructing low-degree parametric surfaces from triangular meshes which represent the surfaces of 3D objects. The obtained results can be used to calculate accurately, interact objects simulated on the computer, map texture, analyze the physical characteristics of the surface, support computer numeric control (CNC), detect a collision, as well as generate deformation of a surface. This research is necessary, practical and also has many applications, especially the field of product design, VR, RE.

For reconstructing low-degree parametric surface approximating the 3D data points of the given triangular mesh, the thesis proposes using an inverse subdivision scheme to create a control mesh and reduce the degree of a reconstructed parametric surface; as well as applying the geometric approximation method to avoid solving complex linear systems. The main contributions of the thesis in science as follows:

- Propose a solution using the inverse subdivision scheme to simplify the given triangular mesh and use the obtained mesh as control mesh of parametric surface. Therefore, the degree of a reconstructed surface is lower than the one of the methods using the given mesh directly as the control mesh;
- Propose a technique generating knot-vectors over a triangular parametric domain of surface, and then apply for surface reconstruction, as well as improve the accuracy of result surfaces;
- Propose a local approximate method for fitting parametric surface converge to the given triangular mesh, avoid solving complex linear systems. The thesis also proves the convergence of the proposed geometric approximate algorithm;
- Propose a method for reconstructing low-degree parametric surfaces from triangular mesh based on inverse subdivision and geometric approximation algorithm, as well as implement experimental examples to make clear a possibility of the proposed method.

The obtained result of the thesis is that the reconstructed surfaces have low-degree, namely triangular Bézier, B-patch, and B-spline. Most of the current representation models are usually triangular meshes, because of their versatile and flexible properties. On the other hand, the surfaces used in the geometric design are often the low-degree parametric surfaces. Therefore, these research results have practical significance and can be used in many fields: CAD, CNC, RE, VR, FEM, evaluation of physical characteristics of a surface, 3D Data compression, exchange of 3D data over the wireless network environment and on mobile devices.

CHAPTER 1
LITERATURE REVIEW

This chapter presents an overview of the surface models of 3D objects, surface reconstruction methods, analysis and comparison of related research for reconstructing smooth surfaces from polygon meshes; and from there, the approach of the thesis is proposed.

1.1. The surface representation methods

The geometric models used for representing of the 3D object surfaces can be distinguished three main types:

- Polygon mesh;
- Subdivision mesh;
- Parametric surface.

1.2. The model conversion

Both polygon mesh and parametric surface have certain advantages and disadvantages, as well as many practical applications. They have a close relationship. In the process of using, to take advantages of both models, it is an essential domain and has many practical applications to convert between these two models for the same 3D object. The conversion allows objects to represent by a variety of models, which support simulation, compute and analyze physical characteristics, detects collision and deformation, etc.

1.3. The surface reconstruction

Reconstruction of smooth surfaces from the 3D meshes has been applied in science, technology, entertainment,... and has been many studies in recent years. Most of the surface reconstruction studies focus on interpolating or approximating of parametric surfaces or subdivision surfaces to polygon meshes. Therefore, the obtained surfaces are usually the subdivision surfaces or parametric surfaces over the rectangular domain such as Bézier, B-Spline, NURBS, etc.

1.4. The related works

Based on the surface reconstruction method, the studies related to smooth surface reconstruction from 3D meshes can be divided into three groups:

- Interpolation method;
- Approximation method;
- Geometry method.

The limitation of the interpolation method is that it must solve the linear equations system to interpolate the points on a surface, which leads to complexity and high costs. With the approximation method, the reconstructed surface is asymptotically shaped by the data mesh, with better surface quality than the interpolated surface.

Most studies have reconstructed smooth surfaces from the rectangular meshes. Meanwhile, the surface of a real object has free shape. The current approach is to transform the triangular mesh into rectangular one, and then reconstruct the smooth surface from this mesh.

Because the reconstruction process must ensure surface having good qualities, memory optimization and short time; the approach of the proposed method is that using the geometric method to fit the parameter surface to the given triangular mesh and the inverse subdivision scheme to reduce the degree of result surface.

1.5. The proposed approach

Based on understanding, synthesizing, analyzing the knowledge and related studies; and take advantages of the triangular mesh and the surface over the triangular parameter domain, as well as the benefits of the inverse subdivision scheme to reduce the degree of reconstructed parameter surface; the proposed method in this thesis is that modeling the low-degree parameter surfaces from the triangular meshes based on *the inverse subdivision scheme* and *the local geometry approximation technique*.

By using the inverse subdivision scheme to simplify the initial mesh, and then employing the points of the obtained mesh as the control points of the parameter surface; the result is that the degree of reconstructed parameter surface will be reduced and lower than the use of original mesh as control mesh. As a result, the obtained surfaces are smooth, avoid the unwanted undulation of high-degree parametric surfaces.

In addition, the proposed model uses the local geometric approximation method for fitting the parameter surface converged gradually to the data points of the original triangular mesh through a number of iterations. The surface approximation is performed visually and accurately until the reconstructed surface passes through most of the original data points.

1.6. The conclusion of chapter 1

Based on the advantages of 3D geometric models and geometric method, the thesis proposes a reconstruction method for the low-degree parameter surface using *the inverse subdivision scheme* and *the local geometric approximation technique*.

In Chapters 2 and 3, the thesis will delve into the presentation and processing of objects related to research direction such as subdivision and inverse subdivision of triangular mesh, surfaces over triangular parameter domain,... And then proposing a method of generating control mesh based on inverse subdivision scheme, introducing a technique of constructing the surface over triangular parameter domain; As a basis for proposing a model of parametric surface reconstruction from the triangular mesh in Chapter 4.

CHAPTER 2
GENERATING CONTROL MESH OF PARAMETRIC SURFACE BASED ON INVERSE SUBDIVISION

For reconstructing the low-degree parameter surfaces from the triangular meshes, this chapter presents subdivision scheme, compares schemes to determine appropriate subdivision scheme. And then, the inverse subdivision scheme is determined, and a method for generating control mesh of the parameter surface based on the inverse subdivision scheme is proposed.

2.1. The subdivision on a triangular mesh

The subdivision allows representing multi-resolution surfaces with free-form topology. The emergence of subdivision in the surface simulation field has created a new breakthrough in CG, animation, 3D games,... For applications that require only rendering smooth surface, the subdivision mesh is an appropriate choice to replace the parameter surface.

Recognizing Loop subdivision has outstanding advantages. Because it is an approximating subdivision, the triangular mesh obtained after subdivision process will tend to shrink compared to the mesh in the previous subdivision step. This also means that if the inverse subdivision is applied to a triangular mesh, then the mesh obtained after each inverse subdivision step will tend to bulge and become the convex hull of this triangular mesh. This finding is very important for selecting the Loop inverse subdivision in the approach to use the inverted mesh as the control mesh of the reconstructed parameter surface.

The Loop subdivision is an approximating face-split scheme based on splines over a triangular parametric domain, which produces C^2-continuous triangular B-splines surfaces over a regular mesh. In each step of this subdivision, each triangular face of a coarse mesh is split into four smaller triangular faces. After each step *i* of Loop subdivision, the set of vertices of meshes M^i includes two types:

- Vertex-vertices: old vertices are modified;
- Edge-vertices: new vertices are inserted into the edges.

After each step *i* of Loop subdivision, the position of vertex-vertices p^{i+1} is determined as follows:

$$p^{i+1} = \alpha p^i + \beta \sum_{j=1}^{l} p_j^i \qquad (2.1)$$

And the position of edge-vertices p_1^{i+1}, p_2^{i+1} … p_k^{i+1} is determined as follows:

$$p_j^{i+1} = \frac{3}{8} p^i + \frac{3}{8} p_j^i + \frac{1}{8} p_{j+1}^i + \frac{1}{8} p_{j-1}^i \qquad (2.2)$$

with j=1..l and l is the valence of the vertex p^i.

2.2. The inverse subdivision on a triangular mesh

The inverse subdivision aims at constructing a coarse mesh from a given dense mesh. Although the inverted mesh has data less, it is still in the geometry and topology and can be completely recovered by using the inverse subdivision scheme. By this way, it saves storage space, reduce bandwidth, suit for graphics applications on mobile and represent multi-resolution surfaces depending on the viewpoint.

In particular, if the Loop inverse subdivision is applied, the data point number of the obtained mesh will reduce and tend to bulge comparing the mesh before inverted subdivision. Thus, the inverted mesh is well suited for reconstructing parametric surface by using this coarse mesh as the control mesh, as well as the convex hull of the surface. By this way, it avoids working on higher order polynomials, reduces bumpiness of the reconstructed parametric surface.

Letting μ is a weight of vertex p^{i+1} and η is a weight of its neighbor vertices, the position of vertex p^i is determined as follows:

$$p^i = \mu.p^{i+1} + \eta.\sum_{j=1}^{l} p_j^{i+1} \tag{2.3}$$

with

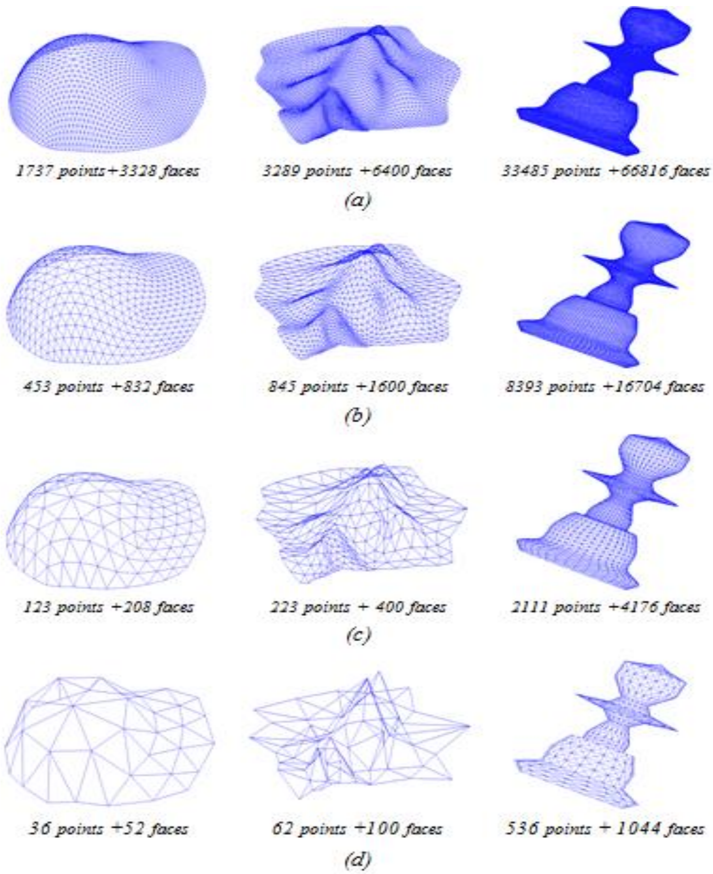
$$\mu = \frac{5}{8\alpha - 3} \quad và \quad \eta = \frac{\alpha - 1}{l\left(\alpha - \frac{3}{8}\right)} \tag{2.4}$$

2.3. Generating control mesh of parametric surface based on inverse subdivision

For reconstructing the low-degree parametric surface from the triangular mesh, the proposed approach is based on the inverse subdivision and uses the Loop inverse subdivision scheme to generate the control mesh of the parametric surface. The reconstructed surfaces over the triangular parametric domain include triangular Bézier, B-patch, and triangular B-spline. Therefore, the original triangular mesh must be adjusted to match the conditions of the subdivision mesh and the control mesh of the surface over the triangular parameter domain.

For meeting these conditions, the thesis proposes the process of constructing the control mesh from the triangular mesh as follows:

- Adjusting the triangular mesh to suit the control mesh of surface;
- Updating the data structure to match the subdivision mesh;
- Simplifying the obtained mesh by using the Loop inverse subdivision scheme.



Hình 2.34. The original mesh (a) and obtained meshes after 1, 2, 3 steps of the inverse Loop subdivision (b,c,d)

Figure 2.34 illustrates the obtained results after applying the inverse Loop subdivision scheme to the triangular meshes, namely *baseball cap*, *mountain*, and *pawn*. From the original meshes (Fig. 2.34a), applying the Loop inverse subdivision scheme, after $i = 1, 2, 3$ steps of inverse subdivision (Figure 2.34bcd), the original meshes have been simplified, and their size decreases exponentially after each step of the inverse subdivision. In particular, the number of faces decreased to only one-quarter of the mesh at the previous step of the inverse subdivision process. The obtained coarse mesh sizes are small. The number of vertices, faces, and edges reduced sharply but still ensure the geometry and topology of the meshes. Moreover, it is possible to restore the original mesh by applying the Loop subdivision scheme.

With this result, the thesis will use the inverted mesh as the control mesh of the surface for reconstructing the surface over the triangular parameter domain, and this reconstruction will be presented in Chapter 4 of the thesis.

2.4. The conclusion of chapter 2

Based on research, analysis, comparison and experimental implementing; the thesis realizes that the Loop subdivision scheme has many advantages and suit for using its inverse subdivision to simplify the control mesh and reduce the degree of the reconstructed parametric surface. From the constructed inverse subdivision scheme, the thesis has applied this scheme to create the control meshes of the surfaces. In addition, for the original triangular meshes to satisfy the conditions of the control mesh and the subdivision mesh, the original meshes need to be modified and updated these data structure before inverting subdivision. The results of this chapter will be used to construct triangular Bézier, B-patch and triangular B-spline surfaces in Chapter 3.

CHAPTER 3
SURFACE OVER TRIANGULAR PARAMETRIC DOMAIN

The contents of this chapter focus on the presentation, geometric properties, position evaluation of a point on the parametric surface. A modification technique of knot-vectors on the parameter domain is also given at the chapter end.

3.1. Related Concepts

This section describes mathematical concepts of the parametric surface representation such as: barycentric coordinates, blossoming, half-open convex hull,…

3.2. Surfaces over triangle parameter domain

The surface over a triangle parameter domain has its control mesh is the triangular mesh which is a common representation form and allows modeling the object surface with arbitrary shape. Therefore this surface has many advantages in simulating the surface of a 3D object on a computer. On the other hand, because Bernstein polynomial on the triangular parameter domain is more complex than the Bernstein univariable polynomial and the B-spline over the rectangular parameter domain, the surface on the triangular parameter domain is still an interesting research matter and has many applications. The thesis purpose is to reconstruct the parameter surfaces from the triangular meshes thus the next section will focus on these surfaces.

The triangular Bézier is the parameter surface with the polynomial function of the parameter *u* and defined on the parameter domain in triangle form. The polynomial degree depends on the number of control point used to define the surface. The triangular Bézier surface does not pass through the control points. It is in the convex hull, which is created by these points. Moreover, thanks to the control points, its shape can be easily adjusted, which is why the Bézier surface is commonly used.

The B-patch surface is based on the idea of "knot pulling " at three angles on the parameter domain of the triangle Bézier surface. The knot set corresponding to each corner of the parameter domain is a knot-vector. Similar to the control points, knot-vectors play an important role in adjusting the surface shape.

The triangular B-spline surface, also known as DMS-splines, is a combination of both smoothness of simplex spline and local control of B-patch surface. The n degree triangular B-spline is defined by linear combinations of simplex splines. This surface is automatically continuous C^{n-1}, without connecting between patches.

3.3. Determining knot-vectors on triangular parameter domain

For the B-patch and triangular B-spline, their shape depends not only on the location of the control points but also on the configuration of the knot-vectors on the parameter domain. In this section, the thesis proposes how to locate knots in knot vectors on the parameter domain of the surfaces having degree 2, 3 and 4. It then supports the surface construction and the knot-vector adjustment in the geometric approximation process, which is used in the proposed model for reconstructing the parametric surfaces in Chapter 4 of the thesis.

3.4. The conclusion of chapter 3

The triangular Bézier, B-patch, and triangular B-spline have many advantages, suit for a surface having free–shape. Therefore, the thesis will use these surfaces to reconstruct the 3D object surfaces from triangular meshes on the computer. The surface shape is not only influenced by the location of the control points but also depends on the configuration of knot vectors on the parameter domain. Therefore, in this chapter, the thesis also proposes how to locate the knots of the knot-vector. This result will be used for constructing surfaces and adjusting their shape during the reconstruction process of the parametric, which will be proposed in chapter 4 of this thesis.

CHAPTER 4
RECONSTRUCTION OF PARAMETRIC SURFACE BASED ON INVERSE SUBDIVISION SCHEME

Based on the obtained results in Chapters 2 and 3, in this chapter, the thesis proposes a model for reconstructing parametric surfaces from the triangular meshes by using the Loop inverse subdivision and applying local geometry approximation method. The result triangular Bézier, B-patch, and triangular B-spline surfaces have low-degree and approximate to the original data meshes.

4.1. Reconstructing parametric surface

4.1.1. Introduce

Triangular meshes are commonly used to simulate 3D objects on the computer because they allow representing surfaces with free-shape and fast processing speed. Thus, the thesis focuses on reconstructing parameter surfaces from data points of the given triangle meshes

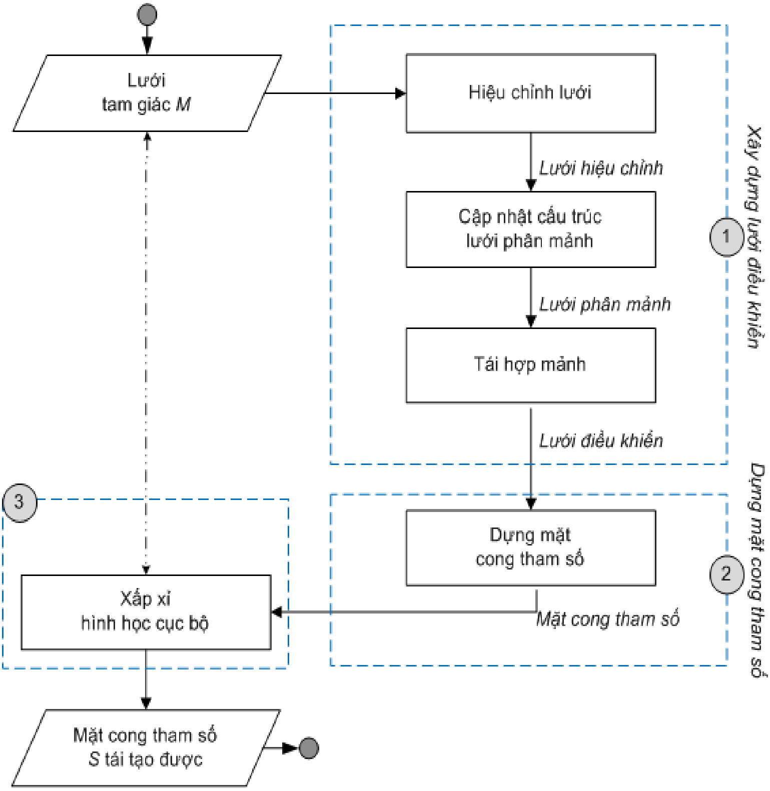
The problem: Given triangle mesh *M*, which formed by m data points $p_j|_{j\ =\ 1..m}\ \in R^3$. Reconstruct a surface S over triangular parametric domain approximating this mesh *M*.

4.1.2. The proposed method

Based on the Loop inverse subdivision scheme on the triangular mesh and the geometric approximation method, the thesis proposed a model for reconstructing the low-degree parametric surface *S* (namely, the triangular Bézier, B-patch, triangle B-spline) from the triangle mesh M (Fig. 4.1). This model consists of the following main steps:

- Step 1: *Edit triangle mesh*. From the initial mesh *M*, this mesh will be adjusted to match the control mesh by adding a few vertices to enough points of the control mesh. In addition, for increasing the quality of the mesh, the weak triangles of the grid are reversed to meet Delaunay conditions;
- Step 2: *Update the data structure*. *M* mesh is updated data structure to become subdivision mesh M^0.
- Step 3: *Simple mesh using inverse subdivision*. By applying the inverse Loop subdivision scheme, the M^0 triangular mesh will be simplified. After some step of the inverse subdivision, it becomes a coarse M^i mesh;
- Step 4: *Construct the parameter surface*. Mesh M^i will be used as control mesh to construct the S^i surface. Thus, the reconstructed parameter surface will have a lower degree than the original mesh as the control mesh;
- Step 5: *Geometric approximation*. The S^i surface is updated by adjusting the position of its control points. After each step of fitting,

the S^i surface will be fitted back to the data points of the original mesh *M*.



Hình 4.1.Reconstruction of the parametric surface using inverse subdivision.

There are three main stages in the proposed model:

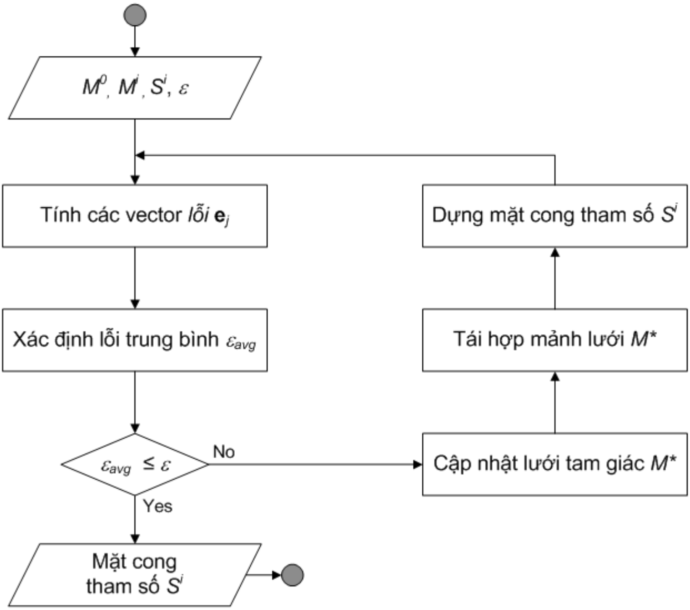
- Stage 1: *Generating control mesh*. It consists of steps 1, 2 and 3. The purpose of stage 1 is to modify the original triangular mesh and to create the control grid of the parameter surface. The steps of this stage are detailed in Section 2.3, Chapter 2 of the thesis;

- Stage 2: *Constructing parametric surface*. The purpose of this stage is to construct the surface over the triangular parametric domain with the control mesh obtained at stage 1. Stage 2 corresponds to Step 4 of the proposed model. The work of Stage 2 is also presented in Section 3.3, Chapter 2 of the thesis;

- Stage 3: *Geometric approximation*. At each step of fitting, this stage produces a parametric surface. This surface is compared to the local deviation at each data point to determine the accuracy of the displacement. This stage is also Step 5 of the proposed model and will be further clarified in this chapter.

4.1.3. Geometric approximation technique

Letting M^0, M^i and S^i are the given meshes and the obtained parameter surfaces after completing Step 4 in the proposed model shown in Figure 4.1, ε is the tolerance. The proposed local geometry approximation method is shown in the diagram in Figure 4.3.



Hình 4.3. Geometric approximation technique

In the process of geometry approximation, a series of triangular meshes M^* is created, and these meshes are simplified into meshes M^i. Correspondingly, a sequence of parameter surfaces is also generated. These surfaces tend to converge towards the original mesh. The fitting process halts when an average error ε^i_{avg} is less than tolerance ε. Finally, the reconstructed parameter surface will be approximated to the initial data points with the smallest mean error.

4.1.4. Convergence of geometric approximation method

Based on the proposed geometry approximation technique, this section analyzes the convergence of the reconstructed parameter surface compared to the original data points.

4.2. Reconstructing triangular Bézier surface

Based on the proposed model, this section provides a geometric approximation algorithm to reconstruct the low-degree triangular Bézier along with some experimental results to prove the feasibility of the proposed method.

4.3. Reconstructing B-patch surface

Based on the proposed model, this section provides a geometric approximation algorithm to reconstruct the low-degree B-patch along with some experimental results to prove the feasibility of the proposed method.

4.4. Reconstructing triangular B-spline surface

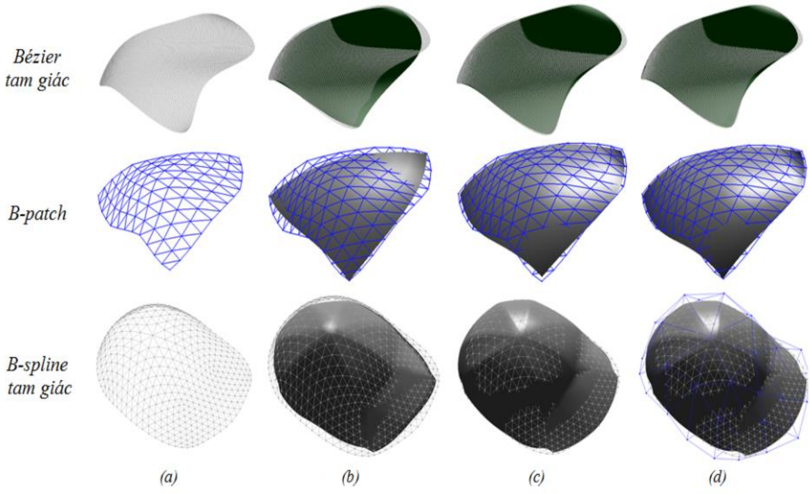
Based on the proposed model, this section provides a geometric approximation algorithm to reconstruct the low-degree triangular B-spline along with some experimental results to prove the feasibility of the proposed method.

4.5. Overall assessment of experimental results

Information on the initial mesh used for testing and the result Bézier, B-patch and B-spline surface models are shown in Table 4.5.

Bảng 4.5. Models of testing

Mô hình thực nghiệm		*Bézier*	*B-patch*	*Bspline*
Lưới ban đầu	*Số điểm*	4753	91	453
	Số mặt	9216	144	832
Thông số tính toán	i	2	2	2
	k	10	10	10
	Thời gian (s)	< 1	12	68
Mặt cong kết quả	*Mặt cong*	*Bézier*	*B-patch*	*B-spline*
	Bậc	24	3	2
	Số đỉnh đ.k	325	10	36
	Số mặt	575	9	13



Hình 4.15. Reconstruct triangular Bézier, B-patch and triangular B-spline: (a) original mesh, (b,c,d) obtained surfaces after k = 1,5,7 steps of fitting.

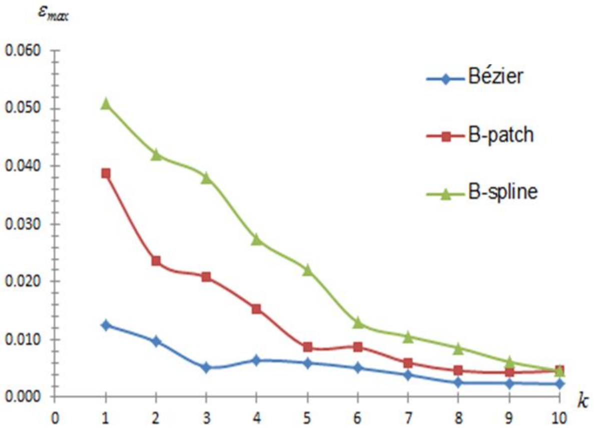
Figure 4.15 illustrates the initial meshes and obtained surfaces after k = 1.5,7 steps of fitting in the geometric approximation process. We can recognize that after the first step k = 1, the difference between the obtained parameter surfaces and the original meshes is quite large, especially the boundary of the surfaces and the boundary curves of the origin meshes. However, after k = 5,7 steps of fitting, the boundary of the surface is gradually fitted to the boundary of the original mesh. At the same time, the rest area of the surface is "stretched" and translocated to the data points. The difference in the obtained surfaces from the original meshes was also reduced at the steps k = 5,7. By applying i = 2 times of inverse subdivision, the obtained surfaces have a much lower degree than using the original mesh as the control mesh. The reconstruction time is very short. Although it is necessary to determine the simplex-spline to calculate the coordinates of each point on the surface, it only takes about 1 minute to reconstruct the triangle B-spline.

Table 4.6. The parameters of deviation and convergence

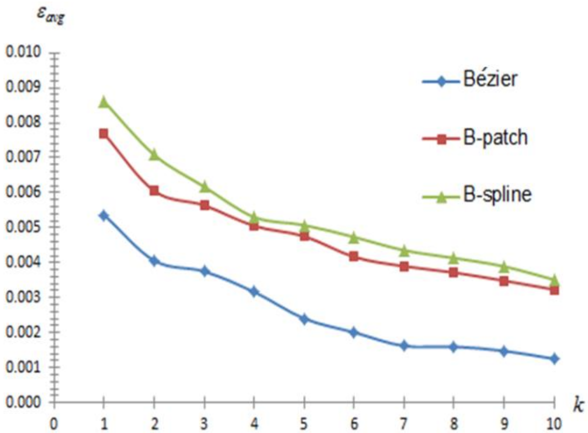
k	Bézier tam giác			B-patch			B-spline tam giác		
	ε_{max}	ε_{avg}	N_ε	ε_{max}	ε_{avg}	N_ε	ε_{max}	ε_{avg}	N_ε
1	0.012535	0.005366	68.869	0.038759	0.007678	41.340	0.050843	0.008605	54.736
2	0.009669	0.004061	79.749	0.023724	0.006053	63.013	0.042221	0.007084	74.096
3	0.005288	0.003751	87.790	0.020769	0.005641	69.739	0.038096	0.006174	82.127
4	0.006369	0.003168	95.083	0.015311	0.005053	76.889	0.027457	0.005305	85.873
5	0.006002	0.002406	95.972	0.008753	0.004758	81.810	0.022018	0.005075	87.022
6	0.005086	0.002018	96.828	0.008667	0.004166	87.157	0.012983	0.004735	89.266
7	0.003902	0.001623	98.567	0.006003	0.003900	88.918	0.010551	0.004356	90.549
8	0.002548	0.001602	96.289	0.004603	0.003720	90.464	0.008579	0.004136	91.004
9	0.002509	0.001482	96.319	0.004283	0.003487	93.771	0.006159	0.003895	91.480
10	0.002316	0.001253	97.956	0.004648	0.003234	93.971	0.004591	0.003516	92.417

Table 4.6 shows the parameters of deviation and convergence during reconstruction through k = 10 first steps of fitting. We can recognize that the maximum deviations ε_{max} and the mean deviation ε_{avg}

are inversely proportional to the number of fitting steps k. Meanwhile, the level of convergence N_ε is proportional to k. This shows that after a few steps of fitting, the obtained parameter surfaces the converge toward the initial mesh points.



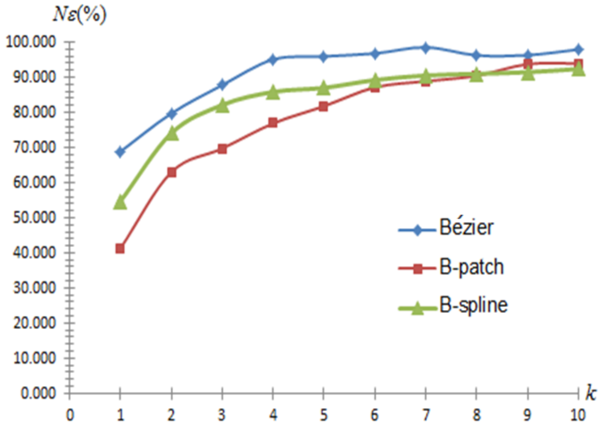
Hình 4.16: Maximum deviations with respect to the number of iteration.



Hình 4.17: Average deviations with respect to the number of iteration

Figures 4.16, 4.17 and 4.18 show the convergence of parametric surfaces in the geometric approximation process. The maximum deviation ε_{max} and the mean deviation ε_{avg} in all three models strongly depend on the number of fitting steps k. These values decreased sharply during the first five steps (Figures 4.16 and 4.17), then stabilized at the remaining steps and ranged from 0.002 to 0.004 (for ε_{max}) and 0.001 to 0.003 (for ε_{avg}). This shows that the obtained parametric surfaces reach fast convergence on the data points after just a few geometric fitting.

In contrast, the graphs in Figure 4.18 show that the number of iterations k increases, the higher the convergence N_ε. Convergence N_ε increased sharply in the first three steps, then stabilized and gradually reached 92% (for triangular B-spline), 94% (for B-patch) and 98% (for triangular Bézier surface).



Hình 4.18: Convergence with respect to the number of iteration

The triangular Bézier surface gives better results than the other two surfaces, with lower deviations and higher convergence. This can explain that because the parameter domain of the triangle Bézer is only a domain triangle, there are no knot-vectors at the corner points. Whereas, with B-patch and B-spline surfaces, the triangular parameter domain have knot vectors at the points. Therefore, similar the position of the control points, the configuration of the knot-vectors also partially affects the result of the surface reconstruction.

4.6. The conclusion of chapter 4

In this chapter, based on the inverse subdivision scheme and the local geometric approximation method, the thesis proposed a model for reconstructing the low-degree parameter surface from a triangular mesh. The three stages of the proposed model include: generating control mesh, constructing surface, and geometric approximation. The first two phases are presented in chapters 2 and 3 respectively. The geometric approximation phase is detailed in this chapter, and the convergence of the proposed method is proved.

Experimental results on triangular Bézier, B-patch, and triangular B-spline surfaces show that, after some geometric approximation steps, the obtained surfaces have degree reduced at least a half, and mesh size reduced by only one-fourth compared to methods which use the original mesh as control mesh. The result parameter surface is passed through most of the data points of the original mesh.

DISSERTATION CONCLUSIONS

1. The achieved content

Through researching and investigating problems related to reconstruction of parameter surfaces from the polygon mesh, the thesis has achieved the following results:

- Analysis and comparison of 3D geometry models to see the pros and cons of each model. In particular, mesh model, subdivision model, and parametric surface model. After that select the triangular mesh, the Loop subdivision scheme and the parametric surface over a triangular domain to solve the given problem;
- Study and evaluate the related research to see the advantages of geometric methods. It is proposed to use the local geometric approximation method to fit the parameter surface gradually to

the triangular mesh to avoid solving the linear system of equations;

- Based on the inverse Loop subdivision scheme and the local geometric approximation method, the thesis proposes a model for reconstructing low-degree parameter surfaces (namely, triangular Bézier, B-patch and triangular B- spline) from triangle meshes;
- Experiment with the proposed model on C ++, OpenGL graphics library, VRML language, ... After that evaluate the results of reconstruction. Three types of parametric surfaces over the triangular domain (namely triangular Bézier, B-patch, and triangular B-spline) have been tested with positive results.

The reconstructed low-degree parametric surfaces are triangular Bézier, B-patch, and triangular B-spline. They allow local controlling the surface shape by adjusting the control points. In particular, the triangular B-spline surface allows representing the 3D object surface with free-shape and automatically continuous between its patches without any connection.

2. Evaluating the obtained results

Based on the inverse subdivision scheme along with the local geometric approximation method, the thesis proposed a new model that allows reconstructing the low-degree surface over triangular parameter domain. The proposed model has the following advantages:

- Reconstructing the low-degree parametric surfaces based on inverse subdivision and geometric approximation. Consequently, it avoids the disadvantages of traditional reconstruction methods, which solve linear systems of equations and least squares approximations;
- By applying the inverse Loop subdivision scheme to the triangular mesh, the obtained surfaces have a much lower level than the use of the original mesh as the control mesh. On the

other hand, the surface approximate to the initial mesh after a few steps of the geometric fitting;

- Reconstructed surfaces are defined on the triangular parameter domain, so they allow the surface representation of the real object to be flexible and to localize the surface through the control points. In particular, the triangular B-splines allow representing smooth global surfaces with any shape.

3. New contributions and further study

Through studying of geometry models describing the 3D object surface and related studies, the thesis has the following contributions:

- Propose an algorithm for modifying triangular mesh based on Delaunay edge flipping and Ruppert's quality mesh generating algorithm. The obtained mesh has better quality than the initial mesh and satisfies the conditions of the control mesh of the surface over the triangular parameter domain;
- Propose a technique for modifying the knot-vectors on the parameter domain to improve the reconstructed surfaces;
- Propose a model for reconstructing low-degree parameter surfaces from the triangular meshes based on the inverse Loop subdivision scheme and the local geometry approximation method;
- Prove the convergence of the proposed method.

However, the research results still have some problems:

- The triangular Bézier and B-patch surfaces are essentially patches and have a triangular shape. Therefore, for representing a free-shape surface, these surfaces need to be connected continuously, this is also a direction for the next study of the thesis;
- Convergence of reconstructing B-patch and B-spline parametric surfaces is not high, about 92% (for B-spline), 95% (for B-patch). This can be explained that adjusting knot-vectors on the parameter domain is not very effective. This is also an open and

challenging problem when researching B-patch and triangular B-spline surface.

Most of the surfaces used in the geometric design are low-degree parametric surfaces, so this result has practical significance in many fields such as CAD, GIS, RE, VR, etc. In addition, it can be applied in fields such as 3D data compression, data exchange on wireless network environment and mobile devices.

From the limitations that can not be overcome, the further research of the thesis is expected as follows:

- Find a solution that connects triangular Bézier surfaces, B-patch surfaces to create a continuous surface with free-shape;
- Find the optimal solution to automatically adjust knot-vectors on the parameter domain of the B-patch and triangular B-spline surfaces to improve the reconstructed results.
- Expand the research direction of surfaces over triangular parametric domain such as simplex-spline, G-patch... and especially on triangular NURBS, an open research direction and many challenges.
- Expand the research direction for reconstructing 3D surfaces from the big data point set by using RBF function.

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