

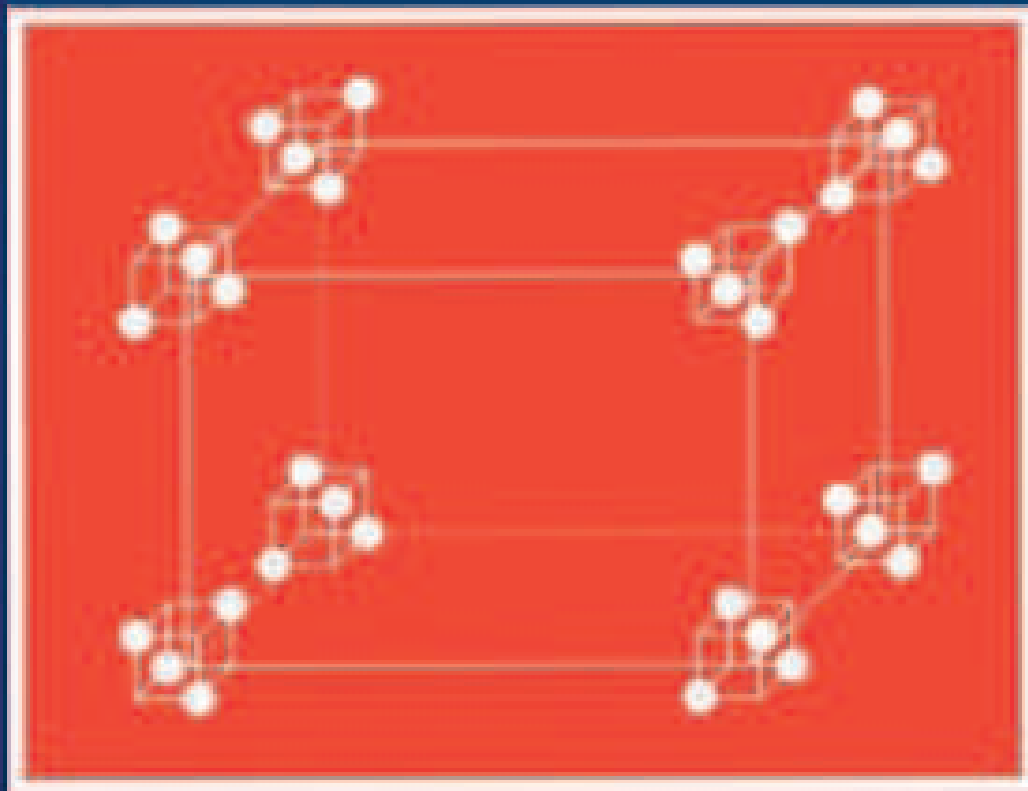
 WILEY

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# Statistical Design and Analysis of Experiments

With Applications to Engineering and Science

## Second Edition



Robert L. Mason   Richard F. Gunst   James L. Hess

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WILEY SERIES IN PROBABILITY AND STATISTICS

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and Science

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Second Edition

Robert L. Mason

Southwest Research Institute  
San Antonio, Texas

Richard F. Gunst

Department of Statistical Science  
Southern Methodist University  
Dallas, Texas

James L. Hess

Leggett and Platt, Inc.  
Carthage, Missouri



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To Carmen, Ann, Janis Sue



# Preface

*Statistical Design and Analysis of Experiments* is intended to be a practitioner's guide to statistical methods for designing and analyzing experiments. The topics selected for inclusion in this book represent statistical techniques that we feel are most useful to experimenters and data analysts who must either collect, analyze, or interpret data. The material included in this book also was selected to be of value to managers, supervisors, and other administrators who must make decisions based in part on the analyses of data that may have been performed by others.

The intended audience for this book consists of two groups. The first group covers a broad spectrum of practicing engineers and scientists, including those in supervisory positions, who utilize or wish to utilize statistical approaches to solving problems in an experimental setting. This audience includes those who have little formal training in statistics but who are motivated by industrial or academic experiences in laboratory or process experimentation. These practicing engineers and scientists should find the contents of this book to be self-contained, with little need for reference to other sources for background information.

The second group for whom this book is intended is students in introductory statistics courses in colleges and universities. This book is appropriate for courses in which statistical experimental design and the analysis of data are the main topics. It is appropriate for upper-level undergraduate or introductory graduate-level courses, especially in disciplines for which the students have had or will have laboratory or similar data-collection experiences. The focus is on the use of statistical techniques, not on the theoretical underpinnings of those techniques. College algebra is the only prerequisite. A limited amount of supplemental material makes use of vector and matrix operations, notably the coverage of multiple linear regression. This material has been placed in appendices and is not essential for an understanding of the methods and applications contained in this book.



The emphasis in this book is on the strategy of experimentation, data analysis, and the interpretation of experimental results. The text features numerous examples using actual engineering and scientific studies. It presents statistics as an integral component of experimentation from the planning stage to the presentation of the conclusions.

This second edition constitutes a significant revision. A number of users of the first edition were surveyed and their feedback was incorporated in the revision. This resulted in deleting some material that wasn't intimately connected to the main thrust of the book, adding some new topics that supplemented existing topical coverage, and rearranging the presentation. For example, some introductory material was eliminated in order to introduce experimental design topics more quickly. A number of new examples were included in several of the chapters. New exercises were added to each of the chapters. In decisions regarding topics, we were guided by our collective experiences as statistical consultants and by our desire to produce a book that would be informative and readable. The topics selected for inclusion in both editions of this book can be implemented by practitioners and do not require a high level of training in statistics.

A key feature of the book, one that was cited as pedagogically beneficial by reviewers, is the depth and concentration of experimental design coverage, with equivalent but separate emphasis on the analysis of data from the various designs. In contrast to the previous edition, however, in the second edition chapters on the analysis of designed experiments have been placed immediately following the corresponding chapters on the respective designs. This was viewed as especially beneficial for classroom use. Instructors and readers can still emphasize design issues in a cohesive manner and can now have the analysis of the data resulting from the use of the respective designs reinforce the important features of the designs by having both the design and the analysis covered in close proximity to one another.

This second edition of *Statistical Design and Analysis of Experiments* is divided into four sections. Part I consists of Chapters 1 to 3 and presents a quick overview of many conceptual foundations of modern statistical practice. These three chapters introduce the reader to the basic issues surrounding the statistical analysis of data. The distinctions between populations or processes and samples, parameters and statistics, and mathematical and statistical modeling are discussed. In addition, elementary descriptive statistics and graphical displays are presented. Throughout the presentation, the informational content of simple graphical and numerical methods of viewing data is stressed.

Chapters 4 to 8 constitute Part II and Chapters 9–13 constitute Part III. These are the heart of the experimental design and analysis portions of the book. Unlike many other statistics books, this book intentionally separates discussions of the design of an experiment from those of the analysis of the resulting data from these experiments. Readers benefit from the reinforcement

of concepts by considering the topics on experimental design in close proximity to one another. In addition, alternatives to the various designs are easily cross-referenced, making the distinctions between the designs clearer. Following the concentrated attention on experimental-design issues, separate chapters immediately provide for the analysis of data from these designs. All too often, texts devote a paragraph to the design of an experiment and several pages to the analysis of the resulting data. Our experiences with this approach are that the material on experimental design is slighted when designs and analyses are presented in the same chapter. A much clearer understanding of proper methods for designing experiments is achieved by separating the topics.

The chapters in Part II concentrate on the design and analysis of experiments with factorial structures. New in the second edition is expanded coverage of statistical graphics (e.g., trellis plots in Chapter 6), three-level and combined two- and three-level fractional factorial experiments (Chapter 7), and expanded coverage on the analysis of data from unbalanced experiments (Chapter 8).

The chapters in Part III stress the design and analysis of data from designed experiments with random factor effects. Added to the second edition is additional material on the analysis of data from incomplete block designs (Chapter 9) and split-plot designs (Chapter 11), new analyses for data from process improvement designs (Chapter 12), and analyses of data from gage R&R studies and data from some designs popularized by Genichi Taguchi (Chapter 13).

Throughout the analysis chapters in Parts II and III, confidence-interval and hypothesis-testing procedures are detailed for single-factor and multifactor experiments. Statistical models are used to describe responses from experiments, with careful attention to the specification of the terms of the various models and their relationship to the possible individual and joint effects of the experimental factors.

Part IV consists of Chapters 14 to 19 and is devoted to the analysis of experiments containing quantitative predictors and factors. Linear regression modeling using least-squares estimators of the model parameters is detailed, along with various diagnostic techniques for assessing the assumptions typically made with both regression and analysis-of-variance models. Analysis-of-covariance procedures are introduced, and the design and analysis needed for use in fitting response surfaces are presented. Identification of influential observations and the concepts of model assessment and variable selection are also discussed.

We are grateful to the Literary Executor of the late Sir Ronald A. Fisher, F. R. S., to Dr. Frank Yates, F. R. S., and to the Longman Group, Ltd., London, for permission to reprint part of Table XXIII from their book *Statistical Tables for Biological, Agricultural, and Medical Research* (6th edition, 1974).

In the first edition, Bea Schube was the John Wiley editor who helped initiate this project, and later Kate Roach was the editor who completed it. We are thankful to both of them as well as to the current Wiley editor, Steve Quigley, for their contributions. For this second edition, we also express our appreciation to Andrew Prince of John Wiley and Joan Wolk of Joan Wolk Editorial Services for their excellent work during the editorial and production process.

We are indebted to many individuals for contributing to this work. Several colleagues read earlier versions of the first edition and made many valuable suggestions on content and readability. We also are thankful to many users of the first edition of this book. Their comments and suggestions, as well as those received from several anonymous reviewers, have been very useful as we developed the second edition.

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