

**NUMERICAL ANALYSIS OF DEGRADATION OF  
CONCRETE STRUCTURES SUBJECTED TO A  
CHLORIDE-INDUCED CORROSION ENVIRONMENT**

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CHLORIDE-INDUCED CORROSION ENVIRONMENT**

by

**Dao Ngoc The Luc**

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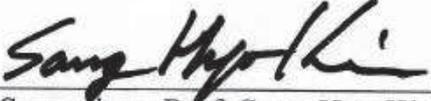
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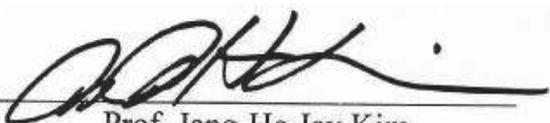
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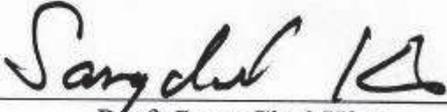
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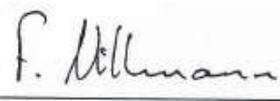
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## NOTATIONS

$\nabla^2$	: Laplacian operator
$\alpha$	: binding constant
$A$	: $\text{Al}_2\text{O}_3$
$\beta$	: binding constant
$\beta_a$	: Tafel slope of the anodic reaction (V/dec)
$\beta_c$	: Tafel slope of the cathodic reaction (V/dec)
$B$	: constant in linear polarization equation (=26mV)
$BC$	: boundary condition
$\bar{C}$	: $\text{CO}_2$
$C$	: $\text{CaO}$
$C_3A$	: tricalcium aluminate
$C_4AF$	: tetracalcium aluminate
$CH$	: calcium hydroxide
$CSH$	: calcium silicate hydrate
$C_b$	: bound chloride
$C_f$	: free chloride
$C_{O_2}$	: concentration of oxygen around the steel (mol/l of pore solution)
$C_{O_2}^S$	: concentration of oxygen at the external surface.
$C_o^S$	: molar concentration of the ion in the consideration in the bulk
$C_R$	: chloride concentration at reinforcement level
$C_S$	: surface chloride concentration
$C_{TH}$	: chloride threshold for corrosion initiation

$\delta$	: thickness of the stagnant layer of electrolyte around the steel surface (= 0.005 mm)
$\delta_{allow}$	: allowable limit value
$d$	: concrete cover thickness
$div$	: divergence operator
$D$	: diffusion coefficient
$D_0$	: diffusion coefficient of original concrete
$D_{28}$	: reference diffusion coefficient at time of 28 days
$D_{O_2}$	: effective oxygen diffusion coefficient in concrete
$D_R$	: diffusion coefficient of repaired concrete
$\eta$	: constant for boundary condition of diffusion problems
$\eta_a$	: anode polarization
$\eta_c$	: cathode polarization
$e_p$	: porosity of the cement paste
$F$	: $Fe_2O_3$
$F$	: Faraday's constant (= $9.65 \cdot 10^4$ C/mol)
$f_{\rho T}$	: factor considering the effect of temperature on concrete resistivity
$f_{\rho S}$	: factor considering the effect of pore degree of saturation on concrete resistivity
$f_{\rho Cl}$	: factor considering the effect of chloride content on concrete resistivity
$FA$	: fly ash
$\gamma$	: a curvature-defining constant
$H$	: $H_2O$
$i_a$	: anodic current density ( $A/mm^2$ )

$i_a^a$	:	anodic exchange current density on active area
$i_{a0}$	:	exchange current density of the anodic reaction (A/mm <sup>2</sup> )
$i_c$	:	cathodic current density (A/mm <sup>2</sup> )
$i_c^a$	:	cathodic exchange current density on active area
$i_c^p$	:	cathodic exchange current density on passive area
$i_{c0}$	:	exchange current density of the cathodic reaction (A/mm <sup>2</sup> )
$i_L$	:	limiting current density of the cathodic reaction (A/mm <sup>2</sup> )
$i_0^T$	:	exchange current density at temperature T
$i_0^{T_0}$	:	exchange current density at reference temperature $T_0$
$k_1$	:	constant for surface chloride
$k_2$	:	constant for shape factor
$\kappa$	:	symmetry factor
$k_o^S$	:	rate constant at standard equilibrium condition
$\lambda$	:	constant for boundary condition of diffusion problems
$\mu$	:	capacity term for diffusion equation ( $\mu = 1 + \alpha\beta C_f^{\beta-1}$ )
$m$	:	a constant accounting for the rate of decrease of diffusion with time
$nnode$	:	number of nodes
$\phi$	:	electrical potential
$\phi^a$	:	electrical potential on active area
$\phi^p$	:	electrical potential on passive area
$\phi_{a0}$	:	equilibrium potential of the anodes
$\phi_{a0}^{T_0}$	:	equilibrium potential of iron reduction at $T_0$ (= -780 mV SCE)

$\phi_{c0}$	: equilibrium potential of the cathodes
$\phi_{c0}^{T_0}$	: equilibrium potential of oxygen reduction at $T_0$ (=160 mV SCE)
$\phi_e$	: equilibrium potential
$R$	: universal gas constant (8.314 J/K.mol)
$\rho$	: electrical resistivity of concrete
$\rho_0$	: concrete resistivity at standard condition
$RH$	: relative humidity
$\bar{S}$	: $\text{SO}_3$
$S$	: slag
$S_p$	: degree of saturation
$SCE$	: Saturated Calomel Electrode
$SF$	: silica fume
$\theta$	: constant for Newmark method
$T$	: absolute temperature (K)
$t_n$	: transference number of all ions in the solution
$T_0$	: reference temperature (=293 K)
$U_\rho$	: activation energy of resistivity
$U_D$	: activation energy of the oxygen diffusion coefficient (kJ/mole)
$w/c$	: water-to-cement ratio
$z_c$	: number of electrons exchanged in the cathodic reaction ( $z_c=4$ )

## ABSTRACT

Annually, billions of dollars are being spent world-wide for the maintenance and repair of deteriorated structures due to chloride attack. Consequently, reliable models for the evaluation of the degradation of these structures are greatly needed.

The degradation of concrete structures subjected to chloride-induced corrosion environment is the result of a complex interaction between many variables that are both time- and space- dependent. Therefore, despite the significant expenditure of much research effort by earlier researchers, currently available models are still limited in their predictive capability and reliability due to their simplifications of various aspects of concrete behavior under chloride attack. The major contributions of the work reported in this thesis can be summarized as follows:

First, a numerical model for chloride penetration into concrete is developed. The newly-developed model is convincingly demonstrated to effectively accommodate the time- and space- dependent chloride transport, chloride binding as well as the effect of steel reinforcement, cracks and the effect of concrete cover replacement/repair; which have not been achieved by earlier numerical models for repaired concrete. Practical implications with regards to repaired concrete exposed to real marine environment are also provided through evaluation of three case studies.

Second, a new inverse relation between current density and potential for the cathodic reaction is proposed. The new inverse relation enables (1) the two nonlinear boundary conditions of potential and current density to be satisfied simultaneously when solving the governing equation and (2) both macro-cell and macro-and-micro-cell modeling to be conveniently solved by a single scheme. Using these findings, a numerical model for simulation of steel corrosion based on adaptive finite element method is developed and its capability is validated through two case studies.

Third, for the first time, the effect of variation in all eight corrosion parameters on the corrosion rate of steel reinforcement in concrete structures is investigated using a numerical model developed in this thesis based on Element-free Galerkin method. Relationships between changes in corrosion rate and changes in each corrosion parameter are presented for both linear and nonlinear regions of the change curves. In addition, observations on the effect of all corrosion parameters and of the anode-to-cathode ratio on corrosion rate are also provided.

Fourth, a numerical model for corrosion-induced cracking of cover concrete, which builds on models for chloride penetration and steel corrosion also developed in this study, is provided. The thesis clearly highlights the significance of taking account of the non-uniform corrosion rust expansion and the corresponding localized corrosion and localized cover cracking. The cover cracking model developed in this study thus offers a significantly better alternative compared with other models that are based on the overly-simplified assumption of uniform corrosion expansion.

Finally, all models developed in the study are incorporated within a reliability-based service life model, which is capable of predicting the remaining service life of concrete structures for three durability limit states (DLS) of corrosion initiation, cover cracking and structural damage in a probabilistic manner. The potential for achieving significantly more economical design is also demonstrated. The unified reliability-based service life model developed in this study forms a solid basis for further development in the effort to realize that potential.

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**Keywords:** concrete, chloride ingress, corrosion, cover cracking, bond strength, finite element, element-free, service life, reliability.