FINAL REPORT

MASTER IN INFORMATION AND COMMUNICATION TECHNOLOGY

By

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Project:

Monogenic Wavelet Transform: Extension to Multispectral Signal

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Abstract

This report develops a monogenic wavelet transform (MWT) with extension to multispectral signals as a new multiscale analysis tool for geometric image features. Monogenic wavelets offer a geometric representation of grayscale images through an AM/FM model allowing invariance of coefficients to translations and rotations. The underlying concept of local phase includes a fine contour analysis into a coherent unified framework. Starting from a link with structure tensors, the XLIM-Icones team proposes a non-trivial extension of the monogenic framework to vector-valued signals to carry out a non-marginal color monogenic wavelet transform. They also give a practical study of this new wavelet transform in the contexts of sparse representations and invariant analysis, which helps to understand the physical interpretation of coefficients and validates the interest of our theoretical construction. A rich feature set can be extracted from the structure multivector, which contains measures for local amplitude, the local orientation, and local phases. Both, the monogenic wavelet transform and the structure multivector are combined with an appropriate scale-space approach, resulting in multi-hyperspectral images.

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Always remind me: Internship topic proposal

Context: In the XLIM-Icones team, they have developed different approaches to the introduction of a color monogenic wavelet transform. Monogenic wavelets offer a geometric representation of grayscale images through an AM/FM model allowing invariance of coefficients to translations and rotations. This yields an efficient representation of geometric structures in grayscale/color images thanks to a local phase carrying geometric information complementary to an amplitude envelope having good invariance properties. So it codes the signal in a more coherent way than standard wavelets.

Objectives: Wavelet based color or multispectral image processing schemes have mostly been made by using a grayscale tool separately on each channel.

In this subject, we propose to discuss definitions that consider a **vector image right** at the beginning of the mathematical definition.

After a general study of the **background of monogenic concept**, the student must study a first approach built from the **grayscale monogenic wavelets** together with a **multiband extension of the monogenic signal** based on **geometric algebra**.

Then, starting from a **link with structure tensors**, the student will build an alternative non-trivial extension of the monogenic framework to vector-valued signals. The crucial point is that the **proposed multispectral monogenic wavelet transform** must be non-marginal as well as it inherits the **coherent geometric analysis** from the monogenic framework.

Finally, the student must address the numerical aspect by introducing an innovative scheme that uses for example a **discrete Radon transform** based on **discrete geometry** (as for the color scheme).

Used Methods and Techniques: Wavelet decomposition, Monogenic concept, Mathematic for the signal, Radon transform, Differential geometry, Numerical aspect of the mathematical decomposition of image.

Contents

List of Special Notations

- 2D coordinates in bold, $\mathbf{x}, \mathbf{u} \in \mathbb{R}^2$ — $\mathbf{x} = [x \;\; y]^\top$ in the spatial domain, $\mathbf{u} = [u \;\; v]^\top$ in the frequency domain
- Euclidean norm: $\|\mathbf{x}\| = \sqrt{x^2 + y^2}$
- Complex imaginary numbers: $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ $\quad \mathbf{i}^2 = \mathbf{j}^2 = \mathbf{k}^2 = -1$
- Real part: $\Re$, Re
- Imaginary part: $\Im$, Im
- Argument of a complex number: $\arg$
- Complex number: $z = \Re\{z\} + \mathbf{i}\Im\{z\} = |z|\, e^{\mathbf{i}\arg\{z\}}$
- Quaternion number: $q = q_0 + q_1\mathbf{i} + q_2\mathbf{j} + q_3\mathbf{k}$
- Quaternion parts: $\mathcal{R}, \mathcal{I}, \mathcal{J}, \mathcal{K}$
- Convolution symbol: $*$
- Hilbert transform: $\mathcal{H}$
- Hilbert transform output: $\mathcal{H}f$ result of the transformation in the spatial domain
- Riesz transform: $\mathcal{R}$
- Analytic signal: f_A
- Riesz transform output: $\mathcal{R}f$ result of the transformation in the spatial domain
- Riesz parts: f_1 and f_2 in the spatial domain
- Monogenic signal: f_M
- Fourier transform: $\mathcal{F}$
- Quaternion Fourier transform: $\mathcal{F}^q$
- Fourier transform: $f \stackrel{\mathcal{F}}{\longleftrightarrow} \hat{f} = \mathcal{F}\{f\}$ means that $\hat{f}$ is the Fourier transform of f
- Hat on a symbol: $\hat{f}$ result of the transformation in the frequency domain
- Dirac delta function: $\delta(t) = \begin{cases} +\infty & \text{if } t = 0 \\ 0 & \text{if } t \neq 0 \end{cases}$
- Isotropic polyharmonic B-spline: β_γ
- Gradient: $\nabla f = \left[\frac{\partial f}{\partial x} \;\; \frac{\partial f}{\partial y}\right]^\top = [f_x \;\; f_y]^\top$
  $= \frac{\partial f}{\partial x} + \mathbf{i}\frac{\partial f}{\partial y} = f_x + \mathbf{i} f_y$
- Laplacian: $\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = f_{xx} + f_{yy}$
- Radon transform output: $D_{\rho,\theta}$ result of the transformation in the spatial domain
- Spectral axis, vector: μ, $\vec{\mu}$ $\quad$ Its modul $|\mu| = 1$ and $\mu^2 = -1$
- Gradient norm: $\mathcal{N}$
- Gradient direction: θ_+
- Subband of multispectral signals: f^{j}

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Chapter 1

Introduction

1.1 Terms and Motivation

Considering continuous signal processing, it is desirable to have tools suitable for audio-visual data thanks in part to their ability to model human perception. For several years, the large topic of defining visually relevant 2D tools gave rise to various geometric wavelet transforms designed to be local in space, direction and frequency. In parallel, the 2D *phase* concept has gained much interest with new definitions for low-level vision and wavelet representations.

Research around phase concept began in the late 40's with the *analytic signal* giving the 1D *instantaneous* phase by using a Hilbert transform. This tool is classical in 1D signal processing. In 2D, the Fourier phase is the first known 2D phase concept, and it has been shown to carry important visual information. Afterwards, study of *phase congruency* proved that the phase can provide meaningful edge detection being invariant to intensity changes. A direct link between local phase and geometric shape of analyzed signal has been clearly established. In optics, image demodulation consists of building a 2D AM/FM model by extracting local amplitude and frequency (derivative of the phase) which in turn appears useful for texture segmentation.

The *monogenic signal* proposed by M. Felsberg is the unifying framework that generalizes the analytic signal carrying out the 2D *AM/FM model*. As well as 2D Fourier atoms are plane waves defined by a 1D sinusoid and an orientation, the most natural 2D phase is basically a 1D phase with a local orientation. The Riesz transform is the key building block to define it - as the proper 2D generalization of the Hilbert transform. Any image is viewed like local plane waves at different scales, with smoothly varying *amplitude*, *phase*, *frequency* and *orientation*. Because the phase concept is meaningful only for narrowband signals, it clearly has to go hand in hand with some multiscale decomposition such as a wavelet transform in order to analyze any class of signal. Among recent propositions of monogenic wavelets, we focus on this since it is tied to a minimally redundant perfect reconstruction filterbank. As we will see, monogenic wavelet coefficients have a directly physical meaning of local 2D geometry.

Differential approaches have a favorable algebraic framework to clearly define true vector tools through the *vector structure tensor*, popularized by Di Zenzo in 1986. These methods are based on estimation of image's gradient and rely on the assumption that resolution is sufficient. Such methods yield remarkable geometric analysis and structure preserving regularization of color images. We consider the structure tensor based geometric analysis that is intrinsic to the monogenic framework.

It is defined a physical interpretation driven spectral extension of the grayscale monogenic wavelet transform by Unser et al. A few different approaches to wavelet analysis of multi-valued images may be retained in the literature. A vector-lifting scheme is proposed for compression purpose, as well as wavelets within the triplet algebra, but these separable schemes do not feature any geometric analysis, in contrast to our non-separable approach allowing isotropy and rotation invariance. The *multiwavelet* framework yields generalized orthogonal filterbanks for multi-valued signals but seems still limited to non-redundantly sampled filterbanks. The connection with monogenic analysis is not yet apparently contrary to wavelet frames. We have found a quaternionic filterbank for color images based on the quaternion color Fourier transform; we have observed that the quaternion formalism sometimes impedes for properly physical interpretation of the data. The present contribution is a new step in process of works trying to propose a physical/signal form for *multispectral images*. This report will start with the recent definitions around the analytic/monogenic concepts. Then it will consist in proposing new definitions of spectral analytic/monogenic signal. Finally, the non-marginal *spectral monogenic wavelet transform* will be defined together with a practical study of the interpretation and use of *wavelet coefficients*.

1.2 Structure of Report

Chapter 2: It is proposed to discuss features of signal, a split of identity AM/FM representation, preparation for directional Hilbert analysis, a vector image right and hypercomplex filtering for multispectral signals at the beginning of the mathematical definition
Chapter 3: Preparation starts with grayscale monogenic wavelet transforms. Frequently images contain variability in many orientations associated with different components and the MWT complements the orientation scales allow us to isolate individual components and their directionality with a high-decomposition in orientation. The concept of transform phase and amplitude are clarified. A simple form for the magnitude and orientation of the isotropic transform coefficients of a unidirectional signal is derived.